



~ spherical shell.

a.) From chap 24 (Gauss' Law), we know that the field between the shells is:

$$\vec{E} = \frac{kQ}{r^2} \hat{r} \quad \text{for } R_1 < r < R_2$$

So the potential between the shells is

$$\Delta V = V(r=R_2) - V(r=R_1) = - \int_{R_1}^{R_2} \vec{E} \cdot d\vec{s}$$

Choose a radial path: $d\vec{s} = dr \hat{r}$

So:
$$\vec{E} \cdot d\vec{s} = \frac{kQ}{r^2} dr$$

and,

$$\Delta V = - \int_{R_1}^{R_2} \frac{kQ}{r^2} dr = -kQ \left. \frac{r^{-1}}{-1} \right|_{R_1}^{R_2} = \frac{kQ}{r} \Big|_{R_1}^{R_2}$$

$$\Delta V = kQ \left(\frac{1}{R_2} - \frac{1}{R_1} \right) < 0$$

$$\Delta V_c = |\Delta V| = kQ \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

and

Capacitance:
$$C = \frac{Q}{\Delta V_c} = \frac{1}{k} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)^{-1}$$

$$C = 4\pi\epsilon_0 \left(\frac{1}{R_1} - \frac{1}{R_2} \right)^{-1}$$

$$b.) \quad d = R_2 - R_1 = 1.0 \text{ mm} \Rightarrow C = 100 \text{ pF}$$

$$\frac{26-81}{2}$$

$$\text{Now, } R_2 = R_1 + d$$

and

$$C = \frac{1}{K} \left(\frac{1}{R_1} - \frac{1}{R_1 + d} \right)^{-1}$$

$$CK = \left(\frac{R_1 + d - R_1}{R_1(R_1 + d)} \right)^{-1} = \left[\frac{d}{R_1(R_1 + d)} \right]^{-1}$$

So,

$$\frac{R_1(R_1 + d)}{d} = CK$$

or,

$$R_1^2 + dR_1 - CKd = 0$$

put in numbers:

$$R_1^2 + (1 \times 10^{-3})R_1 - 8.99 \times 10^{-4} = 0$$

$$\therefore R_1 = \frac{-1 \times 10^{-3} \pm \sqrt{(1 \times 10^{-3})^2 - 4(-8.99 \times 10^{-4})}}{2}$$

$$= -5 \times 10^{-4} \pm 2.999 \times 10^{-2}$$

$$= \underline{2.949 \times 10^{-2}}, -3.049 \times 10^{-2} \text{ m}$$

$$\text{physical} \Rightarrow R_1 = 2.949 \text{ cm}$$

$$\therefore R_2 = R_1 + d = 3.049 \text{ cm}$$

and

$$D_1 = 2R_1 = \underline{5.898 \text{ cm}}$$

$$D_2 = 2R_2 = \underline{6.098 \text{ cm}}$$