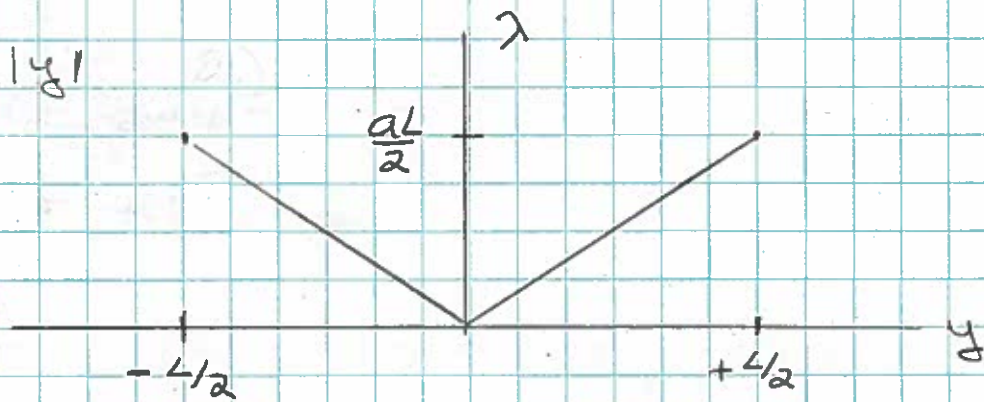


a.) $\lambda = a|y|$



b.) Total charge, $Q = \int_{\text{Rod}} dq = \int_{-L/2}^{L/2} \lambda dy = a \int_{-L/2}^{L/2} |y| dy$

Since $|y|$ is an even function

$$\int_{-L/2}^{L/2} |y| dy = 2 \int_0^{L/2} y dy \quad \text{and } |y| = y \text{ for } y = 0 \rightarrow L/2$$

So: $Q = 2a \int_0^{L/2} y dy = 2a \left(\frac{y^2}{2} \right) \Big|_0^{L/2} = \frac{aL^2}{4}$

So $a = \frac{4Q}{L^2}$

c.) See sketch:

$$\text{at } P: d\vec{E} = \frac{Kdq}{r^2} \hat{r}$$

$$dq = \lambda dy$$

$$r^2 = x^2 + y^2$$

$$\hat{r} = \cos\theta \hat{i} - \sin\theta \hat{j}$$

$$\text{and, } \cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2 + y^2}}$$

So

$$d\vec{E} = \frac{K\lambda dy}{(x^2 + y^2)} \left\{ \frac{x}{\sqrt{x^2 + y^2}} \hat{i} - \frac{y}{\sqrt{x^2 + y^2}} \hat{j} \right\}$$

Now, by symmetry, the y-component integrates to zero.

So:

$$dE_x = \frac{K\lambda x dy}{(x^2 + y^2)^{3/2}} \quad \text{and } x = a|y|$$

$$E_x = \int_{\text{rod}} dE_x = Ka \int_{-L/2}^{L/2} \frac{|y|}{(x^2 + y^2)^{3/2}} dy$$

Now, the integrand is even, so $\int_{-L/2}^{L/2} \rightarrow 2 \int_0^{L/2}$

and $|y| = y$ for $y = 0 \rightarrow L/2$

So:

$$E_x = 2Ka x \int_0^{L/2} \frac{y}{(x^2 + y^2)^{3/2}} dy$$

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Using integral Tables;

$$\begin{aligned} E_x &= 2Ka x \left\{ -\frac{1}{\sqrt{x^2 + y^2}} \right\} \Big|_0^{L/2} \\ &= -2Ka x \left\{ \frac{1}{\sqrt{x^2 + L^2/4}} - \frac{1}{x} \right\} \\ &= 2Ka \left\{ 1 - \frac{x}{\sqrt{x^2 + L^2/4}} \right\} \end{aligned}$$

and $a = 4Q/L^2$

So

$$E_x = \frac{8KQ}{L^2} \left\{ 1 - \frac{x}{\sqrt{x^2 + L^2/4}} \right\}$$