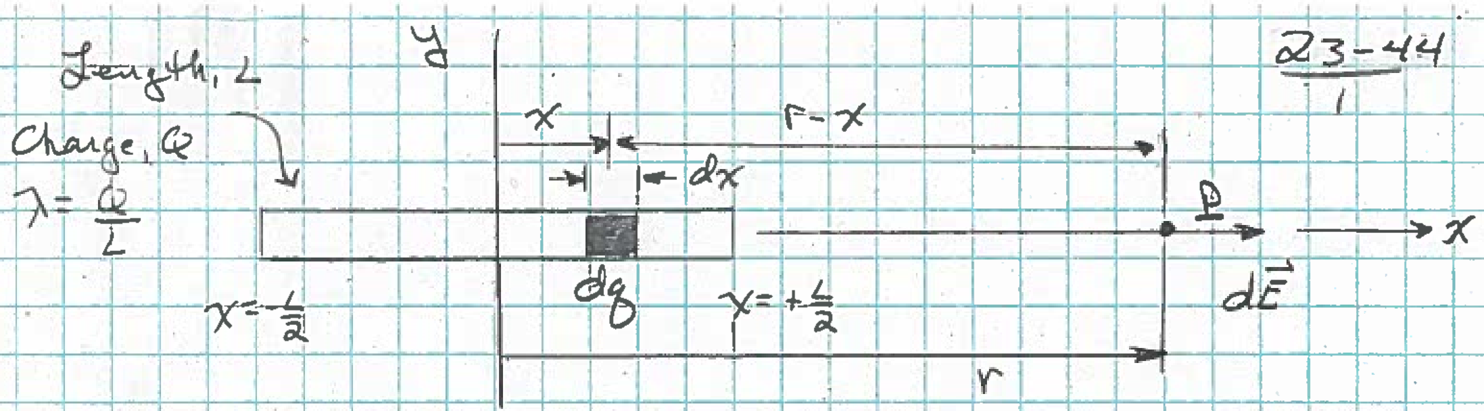

at P: $d\vec{E} = \dfrac{k\,dq}{\left[\begin{smallmatrix}\text{Distance from}\\ dq \to P\end{smallmatrix}\right]^2}\,\hat{r}$

can't use r here since the problem has already used r to locate P.

$$dq = \lambda\,dx$$

$$[\text{Distance } dq \to P] = (r-x)$$

$$\hat{r} = \hat{\imath}$$

So:

$$d\vec{E} = \frac{k\lambda\,dx}{(r-x)^2}\,\hat{\imath}$$

and

$$\vec{E} = \int d\vec{E} = \int \frac{k\lambda\,dx}{(r-x)^2}\,\hat{\imath}$$

$$= k\lambda\hat{\imath}\int_{-L/2}^{+L/2} \frac{dx}{(r-x)^2}$$

Let: $u = r - x \Rightarrow du = -dx$ & limits $\begin{cases} x = -L/2 \Rightarrow u = r + L/2 \\ x = +L/2 \Rightarrow u = r - L/2 \end{cases}$

So $\vec{E} = k\lambda\hat{\imath}\displaystyle\int_{r+L/2}^{r-L/2} \frac{-du}{u^2} = -k\lambda\hat{\imath}\left(-\frac{1}{u}\right)\Bigg|_{r+L/2}^{r-L/2}$

So, $\vec{E} = k\lambda\hat{\imath}\left\{\dfrac{1}{r-L/2} - \dfrac{1}{r+L/2}\right\}$

Now:
$$\vec{E} = K\lambda\hat{r}\left\{\frac{(r+L/2)-(r-L/2)}{(r-L/2)(r+L/2)}\right\}$$
$$= K\lambda\hat{r}\left\{\frac{L}{(r^2-L^2/4)}\right\}$$
$$= \frac{KQ}{L}\left\{\frac{L}{(r^2-L^2/4)}\right\}\hat{r}$$

$$\therefore \quad \vec{E} = \frac{KQ}{(r^2-L^2/4)}\hat{r}$$

b.) for $r \gg L$; $\vec{E} \rightarrow \frac{KQ}{r^2}\hat{r}$ point charge.

c.) $r = 3\,cm$, $L = 5\,cm$, $Q = 3\,nC$
$$\vec{E} = \frac{KQ}{(r^2-L^2/4)}\hat{r} = 9.807\times10^4\,\frac{N}{C}$$