Ideal gas process: $p = cV^{1/2}$

c = constant

a.) W for $V_1 \to V_2$

$$W_{1\to2} = -\int_{V_1}^{V_2} p\,dV = -\int_{V_1}^{V_2} cV^{1/2}\,dV$$

$$= -c\int_{V_1}^{V_2} V^{1/2}\,dV = -c\,\frac{V^{3/2}}{3/2}$$

So:

$$W_{1\to2} = -\frac{2}{3}c\left(V_2^{3/2} - V_1^{3/2}\right) = \frac{2}{3}c\left(V_1^{3/2} - V_2^{3/2}\right)$$

b.) for $n = 0.033$ mol

$T_1 = 150^\circ C = 423\,K$

$V_1 = 300\,cm^3 = 300\times10^{-6}\,m^3$

$V_2 = 200\,cm^3$

$= 200\times10^{-6}\,m^3$

First, find c:

$PV = nRT$ at ① $\Rightarrow P_1 = \frac{nRT_1}{V_1} = cV_1^{1/2}$

So: $c = \frac{nRT_1}{V_1^{3/2}}$

and,

$$W_{1\to2} = \frac{2}{3}\left(\frac{nRT_1}{V_1^{3/2}}\right)\left(V_1^{3/2} - V_2^{3/2}\right)$$

So: $$W_{1\to 2} = \frac{2}{3} nRT_1 \left[1 - \left(\frac{V_2}{V_1}\right)^{3/2}\right]$$

– must use T_1 in K, but V's can be in cm^3!

$$\underline{W_{1\to 2} = 35.2 \text{ J}}$$

c.) find T_2:

$PV = nRT$ at ② $\Rightarrow$ $T_2 = \dfrac{P_2 V_2}{nR} = \dfrac{(cV_2^{1/2})V_2}{nR}$

So: $T_2 = \dfrac{cV_2^{3/2}}{nR} = \left(\dfrac{nRT_1}{V_1^{3/2}}\right)\dfrac{V_2^{3/2}}{nR}$

– using c from part b.

So:
$$\underline{T_2 = T_1\left(\frac{V_2}{V_1}\right)^{3/2} = 230.2\text{ K} = -42.7°C}$$