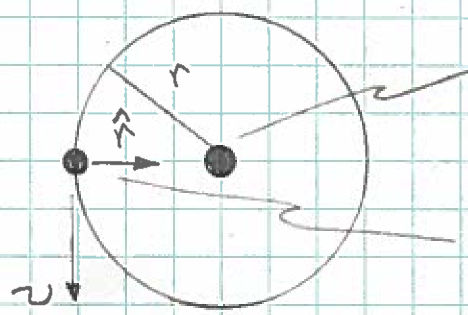
Derivation of Bohr energy levels
(as Bohr did it – not as in text)

Hydrogen



proton
charge $= +e = +1.6\times10^{-19}\,C$

electron
charge $= -e = -1.6\times10^{-19}\,C$
mass, $m = 9.11\times10^{-31}\,kg$

find the total energy of the electron:

For UCM: $\Sigma F_r = \dfrac{K|-e||e|}{r^2} = ma_r = \dfrac{mv^2}{r}$

FBD of e^-

$\odot \xrightarrow{F_E} \quad \xrightarrow{\hat{r}}$

or, $\dfrac{Ke^2}{r^2} = \dfrac{mv^2}{r} \Rightarrow v^2 = \dfrac{Ke^2}{mr}$

Now, the angular momentum postulate:

$L = mvr = n\hbar \Rightarrow v = \dfrac{n\hbar}{mr}$

Put into force eq'n from UCM:

$$\frac{Ke^2}{r^2} = \frac{m}{r}\left(\frac{n\hbar}{mr}\right)^2 = \frac{mn^2\hbar^2}{m^2r^3}$$

So, $\boxed{r_n = \dfrac{n^2\hbar^2}{mKe^2} = n^2 a_B}$

Radii of allowed electron orbits.

$a_B = \dfrac{\hbar^2}{mKe^2} = 0.0529\,nm$

"Bohr Radius"

$\hbar = 1.054\times10^{-34}\,J\cdot s$
$m = 9.11\times10^{-31}\,kg$
$K = 8.99\times10^{9}\,Nm^2/C^2$
$e = 1.6\times10^{-19}\,C$

Now:

Kinetic energy, $KE = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$ from UCM

Potential energy, $U = \frac{k(-e)(e)}{r} = -\frac{ke^2}{r}$

Total energy, $E = KE + U = -\frac{ke^2}{2r}$

Or,

$$E_n = -\frac{ke^2}{2r_n} = -\frac{ke^2}{2}\left(\frac{mke^2}{n^2\hbar^2}\right) = -\frac{mk^2e^4}{2n^2\hbar^2}$$

$\therefore$ $\boxed{E_n = -\frac{E_B}{n^2}}$ where $E_B = \frac{mk^2e^4}{2\hbar^2} = \underline{13.6\,eV}$

energies of allowed electron orbits.

Energy level diagram

⋮

$n=4$ ———— $E_4 = -0.85\,eV$

$n=3$ ———— $E_3 = -1.51\,eV$

$n=2$ ———— $E_2 = -3.4\,eV$

$n=1$ ———— $E_1 = -13.6\,eV$

$$E_n = -\frac{13.6\,eV}{n^2}$$