

Damped Oscillator

$$x(t) = A_0 e^{-\frac{t}{2\tau}} \cos(\omega t + \phi_0) \quad \text{"Damping constant"}$$

where: $m = 250g$, $k = 4.0 \text{ N/m}$, $b = 0.015 \frac{\text{kg}}{\text{s}}$

$x(0) = 0.2 \text{ m}$ & $v(0) = 0 \Rightarrow A = 0.20 \text{ m}$, $\phi_0 = 0$

Now,

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = 3.9999 \approx 4.0 \text{ s}$$

only slightly different than $\sqrt{k/m}$

So,

$$T = \frac{2\pi}{\omega} = 1.571 \text{ s}$$

Now,

$$\tau = m/b = 16.67 \text{ s}$$

and the amplitude decays as:

$$A(t) = A_0 e^{-t/2\tau}$$

So: $\frac{A(t)}{A_0} = \frac{1}{e}$ from the problem.

$$e^{-t/2\tau} = e^{-1}$$

So,

$$-\frac{t}{2\tau} = -1 \Rightarrow t = 2\tau = 33.33 \text{ s.}$$

In this time, the number of oscillation is:

$$\frac{33.33 \text{ s}}{T} = \underline{\underline{22.22}}$$