Differential Equn: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$

Trial Solution:

$$x(t) = A\cos(\omega t + \phi_0)$$

where A, ω, & ϕ_0 are unknown constants.

So:

$$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi_0)$$

&

$$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi_0)$$

Subst. into DE:

$$-A\omega^2\cos(\omega t + \phi_0) = -\frac{k}{m}A\cos(\omega t + \phi_0)$$

So: this is a solution if:

$$\omega^2 = \frac{k}{m} \quad \text{or} \quad \omega = \sqrt{\frac{k}{m}}$$