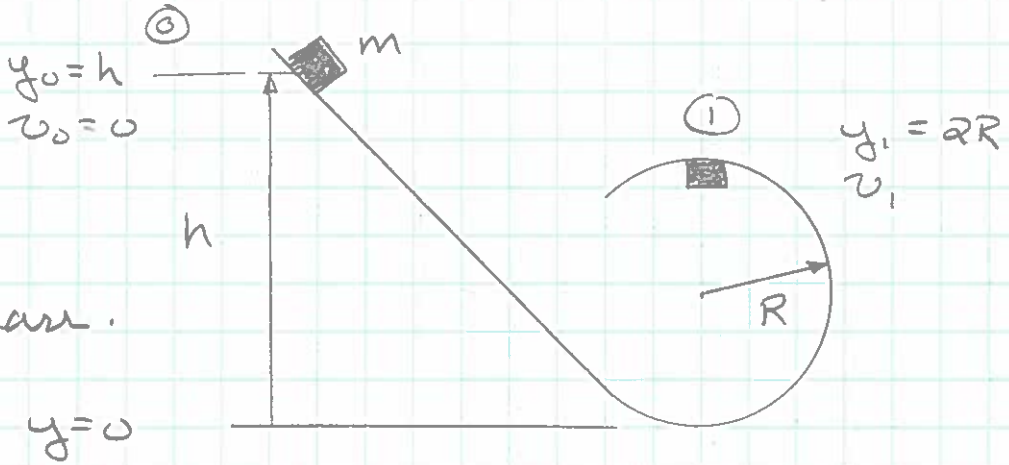


a.) sliding point mass.

First, find v_1 for block to just stay on track.

at 1, FBD

$$\Sigma F_r = w + n = ma_r = \frac{mv^2}{r}$$

To just stay on track, $n \to 0$

$$mg = \frac{mv_1^2}{R} \Rightarrow v_1 = \sqrt{gR}$$

Now, conserve energy, $0 \to 1$:

$$\Delta E_{mech} = \Delta K + \Delta U_g = 0$$

$$\frac{1}{2}m(v_1^2 - v_0^2) + mg(y_1 - y_0) = 0$$

$$\frac{1}{2}v_1^2 + g(2R - h) = 0$$

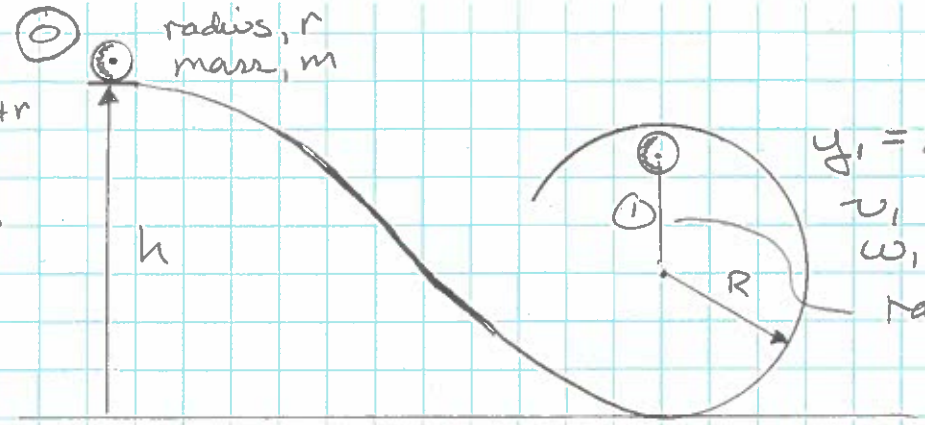
from above, $v_1^2 = gR$

So,

$$\frac{1}{2}gR + 2gR - gh = 0$$

$$\therefore \quad h = \frac{5}{2}R$$

b.)

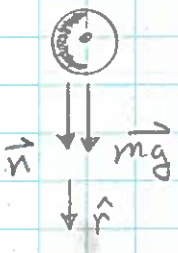


$y_0 = h + r$
$v_0 = 0$
$\omega_0 = 0$

$y_1 = 2R - r$
v_1
ω_1

Rolling Marble.

FBD of marble at ①:



$$\Sigma F_r = n + mg = ma_r = \frac{mv^2}{\text{radius}}$$

To just stay on track, $n \to 0$

So:

$$mg = \frac{mv_1^2}{R-r}$$

or $$v_1 = \sqrt{g(R-r)}$$

Conserve energy $0 \to 1$:

$$\Delta E_{mech} = \Delta K + \Delta U = \cancel{W_{nc}}$$

$$\frac{1}{2}m(v_1^2 - \cancel{v_0^2}) + \frac{1}{2}I_{cm}(\omega_1^2 - \cancel{\omega_0^2}) + mg(y_1 - y_0) = 0$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}I_{cm}\omega_1^2 + mg\underbrace{((2R-r) - (h+r))}_{(2R-2r-h)} = 0$$

Now, for sphere: $I_{cm} = \frac{2}{5}mr^2$ and $\omega = \frac{v}{r}$

So

$$\frac{1}{2}m v_1^2 + \frac{1}{2}\frac{2}{5}m\cancel{r^2}\frac{v_1^2}{\cancel{r^2}} + mg(2R-2r-h) = 0$$

$$\left(\frac{1}{2} + \frac{1}{5}\right)v_1^2 + 2g(R-r) - gh = 0$$

Now, from above: $v_1^2 = g(R-r)$

So:

$$\frac{7}{10}g(R-r) + 2g(R-r) = gh$$

$$\therefore \boxed{h = \frac{27}{10}(R-r)}$$