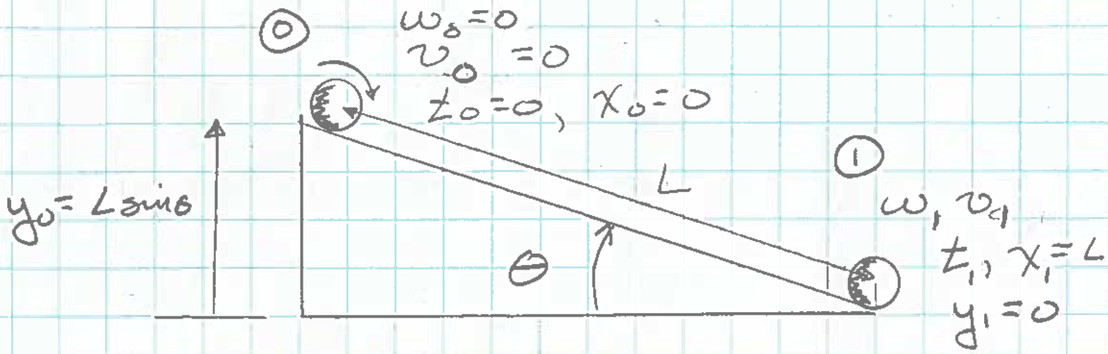

NOTE: $I_{cm} = cMR^2$ c = some number

<u>Conserve energy</u> <u>0 → 1:</u>

$$\Delta E_{mech} = \Delta K + \Delta U_g = 0$$

$$\frac{1}{2}M(v_1^2 - \cancel{v_0^2}) + \frac{1}{2}I_{cm}(\omega_1^2 - \cancel{\omega_0^2}) + Mg(\cancel{y_1} - y_0) = 0$$

and, $I_{cm} = cMR^2$, and $\omega = \frac{v}{R}$

$$\frac{1}{2}Mv_1^2 + \frac{1}{2}(cMR^2)\left(\frac{v_1}{R}\right)^2 - MgL\sin\theta = 0$$

$$\frac{1}{2}v_1^2 + \frac{c}{2}v_1^2 = gL\sin\theta$$

So $v_1 = \sqrt{\dfrac{2gL\sin\theta}{(c+1)}}$

<u>Now</u>: 0 → 1 1D kinematics:

$$v_1^2 = \cancel{v_0^2} + 2a\Delta x \qquad \Delta x = x_1 - x_0 = L$$

$$a = \frac{v_1^2}{2L} = \frac{g\sin\theta}{(c+1)}$$

And, $x_1 = \cancel{x_0} + \cancel{v_0}\Delta t + \frac{1}{2}a\Delta t^2 \qquad \Delta t = t_1 - \cancel{t_0}$

$$L = \frac{1}{2}at_1^2 = \frac{g\sin\theta}{2(c+1)}t_1^2$$

$$\boxed{\text{So: } t_1 = \sqrt{\frac{2(c+1)L}{g\sin\theta}}} \quad \text{or } t_1 = \sqrt{c+1}\sqrt{\frac{2L}{g\sin\theta}}$$

So, time to bottom:

$$t = \sqrt{\frac{2(c+1)L}{g\sin\theta}} = \sqrt{c+1}\sqrt{\frac{2L}{g\sin\theta}}$$

Note for a particle sliding without friction:

$c = 0$ and $t = \sqrt{\frac{2L}{g\sin\theta}} = t_{part.}$

Solid sphere: $I_{cm} = \frac{2}{5}MR^2 \Rightarrow c = \frac{2}{5}$

Spherical shell: $I_{cm} = \frac{2}{3}MR^2 \Rightarrow c = \frac{2}{3}$

Solid Cylinder: $I_{cm} = \frac{1}{2}MR^2 \Rightarrow c = 1/2$

Cylindrical shell: $I_{cm} = MR^2 \Rightarrow c = 1$

So: $t = \sqrt{(c+1)}\, t_{part.}$, so sliding frictionless particle is the fastest.

∴ Solid Sphere: $t = \sqrt{\frac{7}{5}}\, t_{part} = 1.183\ t_{part.}$

Spherical shell: $t = \sqrt{\frac{5}{3}}\, t_{part} = 1.291\ t_{part.}$

Solid Cylinder: $t = \sqrt{\frac{3}{2}}\, t_{part} = 1.225\ t_{part}$

Cylindrical shell: $t = \sqrt{2}\, t_{part} = 1.414\, t_{part}$

∴ Ranking from fastest to slowest:

Solid Sphere
Solid Cylinder
Spherical Shell
Cylindrical Shell.