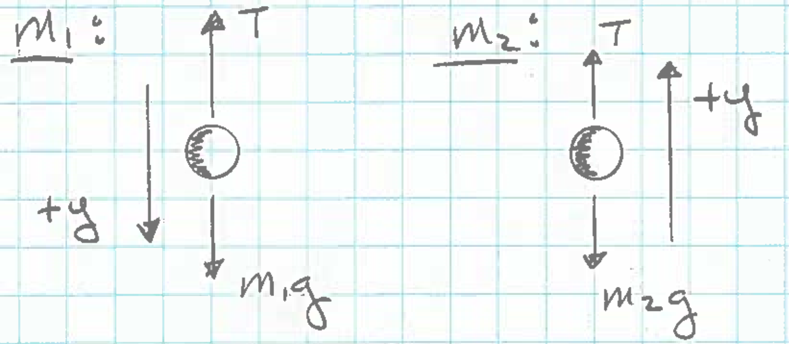
a.) Ideal Massless Pulley:

FBD: Ideal Pulley from chap 7:



$m_1 = 4\ kg$

$m_2 = 2\ kg$

$h = 3\ m$

Constraint: $a_{1y} = a_{2y} = a$

m_1: $\Sigma F_y = m_1 g - T = m_1 a_{1y} = m_1 a$

m_2: $\Sigma F_y = T - m_2 g = m_2 a_{2y} = m_2 a$

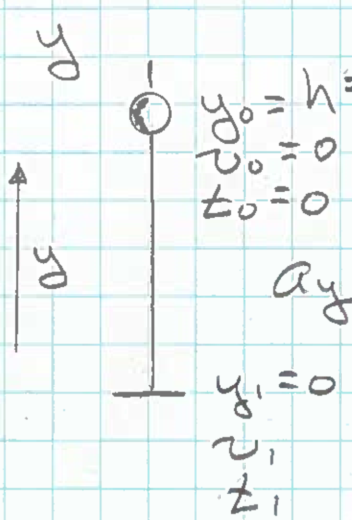
Add equations:

$$m_1 g - T + T - m_2 g = m_1 a + m_2 a$$

$$g(m_1 - m_2) = a(m_1 + m_2)$$

$$a = g\left(\frac{m_1 - m_2}{m_1 + m_2}\right) = 3.267\ m/s^2$$

Kinematics for m_1:



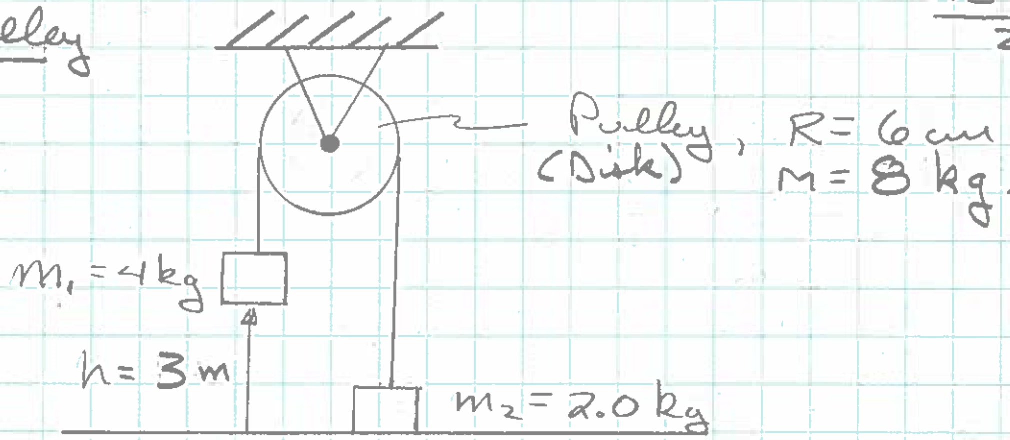
$$a_y = -3.267\ \frac{m}{s^2}$$

$$y_1 = y_0 + v_0 \Delta t + \frac{1}{2} a_y \Delta t^2 \qquad \Delta t = t_1 - t_0$$

$$0 = h + \frac{1}{2} a_y t_1^2$$

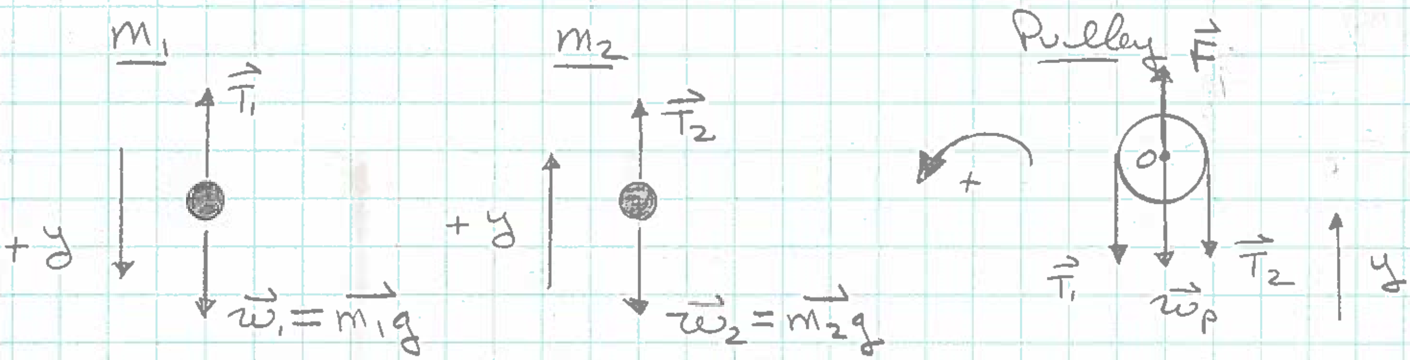
$$t_1 = \sqrt{\frac{-2h}{a_y}} = 1.355\ s$$

b.) Real Pulley



Released from rest; find time for m_1 to hit the floor

FBD's



$\Sigma F_y = w_1 - T_1 = m_1 a_1$ $\qquad$ $\Sigma F_y = T_2 - w_2 = m_2 a_2$ $\qquad$ $\Sigma F_y = F - w_p - T_1 - T_2 = m a_p = 0$

Constraint: $|\vec{a}_1| = |\vec{a}_2| = (|\alpha| R)_{disk} = a$ $\qquad$ $\circlearrowleft + \; \Sigma \tau_o = T_1 R - T_2 R = I\alpha$

NOTE:
$T_1 \neq T_2$ because pulley is not massless.

Force eqn for pulley doesn't really tell us anything.

Acceleration Constraint: $a_1 = a_2 = a$

$$\alpha = a/R$$

Pulley is a Disk: $I = \frac{1}{2} M R^2$

m_1 eqn: $m_1 g - T_1 = m_1 a$ (1)

m_2 eqn: $T_2 - m_2 g = m_2 a$ (2)

Pulley eqn: $T_1 R - T_2 R = \frac{1}{2} M R^2 \frac{a}{R}$

or, $T_1 - T_2 = \frac{M}{2} a$ (3)

3 eqns, 3 unknowns (T_1, T_2, & a)

eqn (1) ⟶ $T_1 = m_1 (g - a)$

eqn (2) ⟶ $T_2 = m_2 (a + g)$

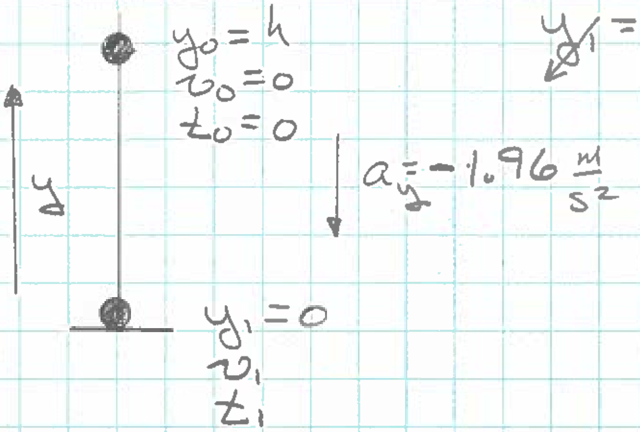
subst. into eqn (3):

$$m_1 (g-a) - m_2 (a+g) = \frac{M}{2} a$$

$$m_1 g - m_2 g = \frac{M}{2} a + m_1 a + m_2 a$$

$$\therefore \quad a = \frac{(m_1 - m_2) g}{\left(\frac{M}{2} + m_1 + m_2\right)} = \underline{1.96 \text{ m/s}^2}$$

Now, do const. accel. kinematics for m_1



$$\not{y_1} = y_0 + \not{v_0 \Delta t} + \frac{1}{2} a_y \Delta t^2 \qquad \Delta t = t_1 - \not{t_0}$$

$$0 = h + \frac{1}{2} a_y t_1^2$$

$$\therefore \quad t_1 = \sqrt{\frac{-2h}{a_y}} = \underline{1.75 \text{ s}}$$

Note $a_y < 0$