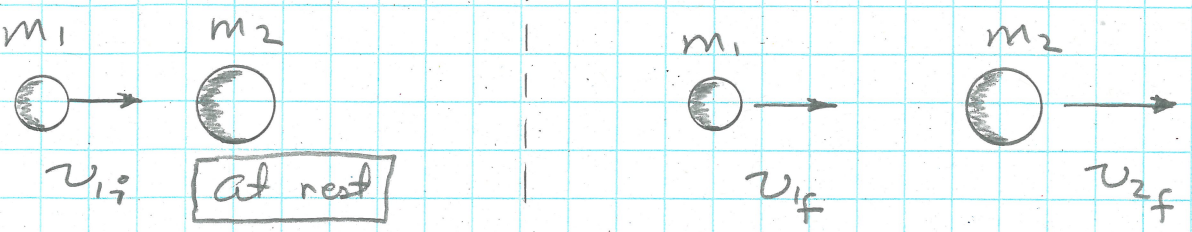
1D Perfectly Elastic Collision



NOTE: v_{1i}, v_{1f}, & v_{2f} are velocity components.

Cons. of momentum: $m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$ (1)

Cons. of KE: $\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$ (2)

Know: masses & v_{1i}; so 2 equ'ns 2 unknowns.

equ'n (1) $\longrightarrow$ $v_{1f} = v_{1i} - \frac{m_2}{m_1} v_{2f}$ (3)

$\longrightarrow$ equ'n (2): $m_1 v_{1i}^2 = m_1 \left(v_{1i} - \frac{m_2}{m_1} v_{2f} \right)^2 + m_2 v_{2f}^2$

So: $m_1 v_{1i}^2 = m_1 v_{1i}^2 - 2 m_2 v_{1i} v_{2f} + \frac{m_2^2}{m_1} v_{2f}^2 + m_2 v_{2f}^2$

or, $0 = -2 v_{1i} v_{2f} + \left(\frac{m_2}{m_1} + 1 \right) v_{2f}^2$

$0 = v_{2f} \left\{ \left(\frac{m_2}{m_1} + 1 \right) v_{2f} - 2 v_{1i} \right\}$

Solution: $v_{2f} = 0$ or $\left\{ \left(\frac{m_2}{m_1} + 1 \right) v_{2f} - 2 v_{1i} \right\} = 0$

So:

$$v_{2f} = \frac{2 v_{1i}}{\left(\frac{m_2}{m_1} + 1 \right)} = \boxed{\frac{2 m_1 v_{1i}}{(m_1 + m_2)} = v_{2f}}$$

Subst. into equ'n (3):

$$v_{1f} = v_{1i} - \frac{m_2}{m_1} \frac{2 m_1 v_{1i}}{(m_1 + m_2)} = \frac{m_1 v_{1i} + m_2 v_{1i} - 2 m_2 v_{1i}}{(m_1 + m_2)}$$

So, $\boxed{v_{1f} = \frac{v_{1i} (m_1 - m_2)}{(m_1 + m_2)}}$