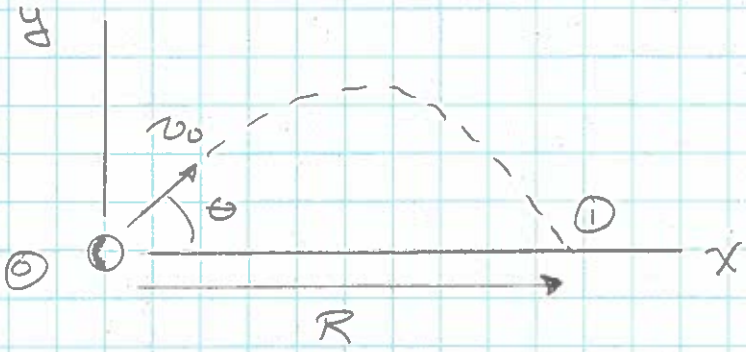
From Chap 4: (no air resistance)

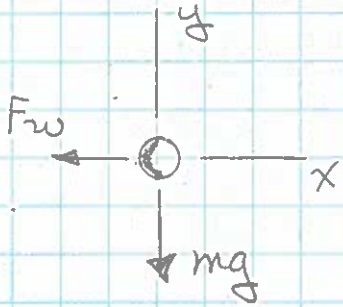


$R = \frac{v_0^2}{g}\sin(2\theta)$ is maximum for $\theta = 45°$

Now: $\vec{F}_w = -F_w\hat{i}$ +headwind force

b.)

FBD



$\Sigma F_x = -F_w = ma_x \Rightarrow a_x = -\frac{F_w}{m}$

$\Sigma F_y = -mg = ma_y \Rightarrow a_y = -g$

Constant acceleration $0 \Rightarrow 1$:

$(x_0, y_0) = (0,0)$ $\qquad (x_1, y_1) = (R, 0)$

$(v_{x0}, v_{y0}) = (v_0\cos\theta, v_0\sin\theta)$ $\qquad t_1$

$t_0 = 0$

y-motion:

$y_1 = y_0 + v_{y0}\Delta t + \frac{1}{2}a_y\Delta t^2 \qquad \Delta t = t_1 - t_0$

$0 = v_0\sin\theta\, t_1 - \frac{1}{2}g t_1^2$

$t_1\left(v_0\sin\theta - \frac{1}{2}g t_1\right)$

So, $v_0\sin\theta - \frac{1}{2}g t_1 = 0 \Rightarrow t_1 = \frac{2v_0\sin\theta}{g}$

x-motion:

$$x_1 = x_0 + v_{x0}\Delta t + \frac{1}{2}a_x \Delta t^2$$

$$R = v_0 \cos\theta\, t_1 - \frac{F_w}{2m} t_1^2$$

Or,

$$= v_0 \cos\theta \left(\frac{2v_0 \sin\theta}{g}\right) - \frac{F_w}{2m}\left(\frac{2v_0 \sin\theta}{g}\right)^2$$

$$= \frac{2v_0^2}{g}\cos\theta\sin\theta - \frac{F_w}{m}\frac{2v_0^2 \sin^2\theta}{g^2}$$

Now, $2\cos\theta\sin\theta = \sin(2\theta)$

So:

$$\boxed{R = \frac{v_0^2}{g}\sin(2\theta) - \frac{2F_w v_0^2 \sin^2\theta}{mg^2}}$$

c.) To find max R, set $\frac{dR}{d\theta} = 0$, solve for θ:

$$\frac{dR}{d\theta} = \frac{v_0^2}{g}\cos(2\theta)\,2 - \frac{2F_w v_0^2}{mg^2}\,2\sin\theta\cos\theta = 0$$

So:

$$\cos(2\theta) - \frac{F_w}{mg}\sin(2\theta) = 0$$

Or,

$$\frac{\sin(2\theta)}{\cos(2\theta)} = \tan(2\theta) = \frac{mg}{F_w}$$

$$\therefore \quad \theta_{max} = \frac{1}{2}\tan^{-1}\left(\frac{mg}{F_w}\right)$$

For example:

for $m = 0.5\,kg$ and $F_w = 0.6\,N$

$$\theta = \theta_{max} = \frac{1}{2}\tan^{-1}\left(\frac{mg}{F_w}\right) = 41.51^\circ$$

The max range without air resistance is

$$R_0 = \frac{v_0^2}{g}\sin(2\theta) = \frac{v_0^2}{g} \quad \text{for } \theta = 45^\circ$$

With air resistance, max range is for $\theta = 41.51^\circ$

$$R = \frac{v_0^2}{g}\sin(2\theta) - 2\frac{F_w}{mg}\frac{v_0^2}{g}\sin^2\theta$$

$$= 0.885\frac{v_0^2}{g}$$

$$\frac{R - R_0}{R_0} = \frac{0.885 - 1}{1} = -0.115$$

$\therefore$ range is reduced by 11.5%