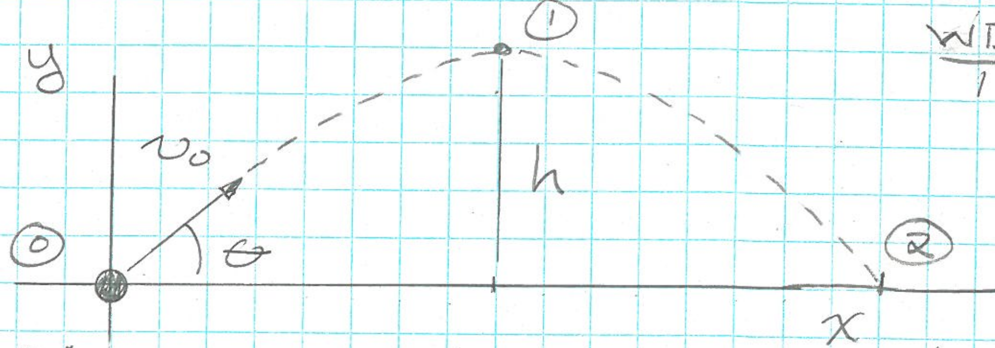

at point 0:

$t_0 = 0$

$(x_0, y_0) = (0, 0)$

$(v_{x_0}, v_{y_0}) = (v_0 \cos\theta, v_0 \sin\theta)$

at point 1:

t_1

$(x_1, y_1) = (x_1, h)$

$(v_{x_1}, v_{y_1}) = (v_{x_0}, 0)$

Note: $a_x = 0$, $a_y = -g$

a.) find height $h = y_1$

y-motion:

$$\cancel{v_{y_1}}^2 = v_{y_0}^2 + 2 a_y \Delta y \qquad \Delta y = y_1 - \cancel{y_0}$$

$$0 = v_0^2 \sin^2\theta - 2 g h$$

$$\therefore \quad h = \frac{v_0^2 \sin^2\theta}{2g}$$

b.) More: How far does it go?

at 2: $(x_2, y_2) = (x_2, 0)$ and t_2

(v_{x_2}, v_{y_2})

find x_2 = "Range"

0→2: y-motion:

$$\cancel{y_2} = \cancel{y_0} + v_{y0}\Delta t + \tfrac{1}{2} a_y \Delta t^2 \qquad \Delta t = t_2 - \cancel{t_0}$$

$$0 = v_0 \sin\theta\, t_2 - \tfrac{1}{2} g t_2^2$$

$$= t_2\left(v_0 \sin\theta - \tfrac{1}{2} g t_2\right)$$

So: $t_2 = 0$ or $v_0 \sin\theta - \frac{g}{2} t_2 = 0$

So $t_2 = \dfrac{2 v_0 \sin\theta}{g}$

Now, x-motion:

$$x_2 = \cancel{x_0} + v_{x_0}\Delta t + \tfrac{1}{2}\cancel{a_x}\Delta t^2 \qquad \Delta t = t_2 - \cancel{t_0}$$

$$= v_0 \cos\theta\, t_2$$

So

$$x_2 = v_0 \cos\theta \left(\frac{2 v_0 \sin\theta}{g}\right)$$

$$= \frac{2 v_0^2}{g} \sin\theta \cos\theta$$

Or, use the trig. identity:

$$\sin(2\theta) = 2 \sin\theta \cos\theta$$

So:

$$R = x_2 = \text{Range} = \frac{v_0^2}{g} \sin(2\theta)$$

"Range Equation"

c.) the range will be maximum when $\theta = 45^\circ$ since $\sin(2 \cdot 45^\circ) = 1$.