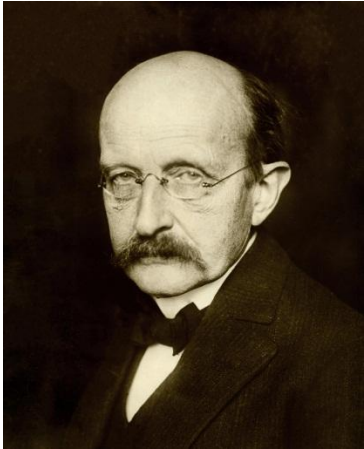
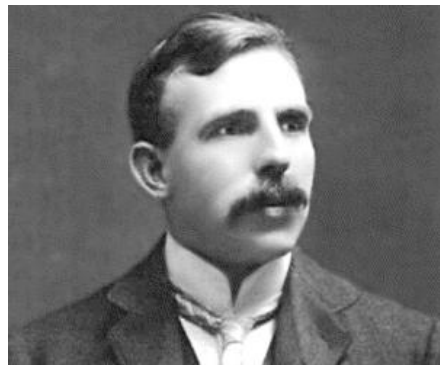


QM-2: A Quick Review up to this point (LC)



In **1900**, **Max Planck** was trying to explain **blackbody radiation**. He modelled the blackbody as a cavity radiator and found that he could successfully reproduce the continuous spectrum only if he assumed that the walls of the cavity emitted and absorbed radiation in integer multiples of discrete quanta with energies given by $E = hf$ where f is the radiation frequency and h is a constant.

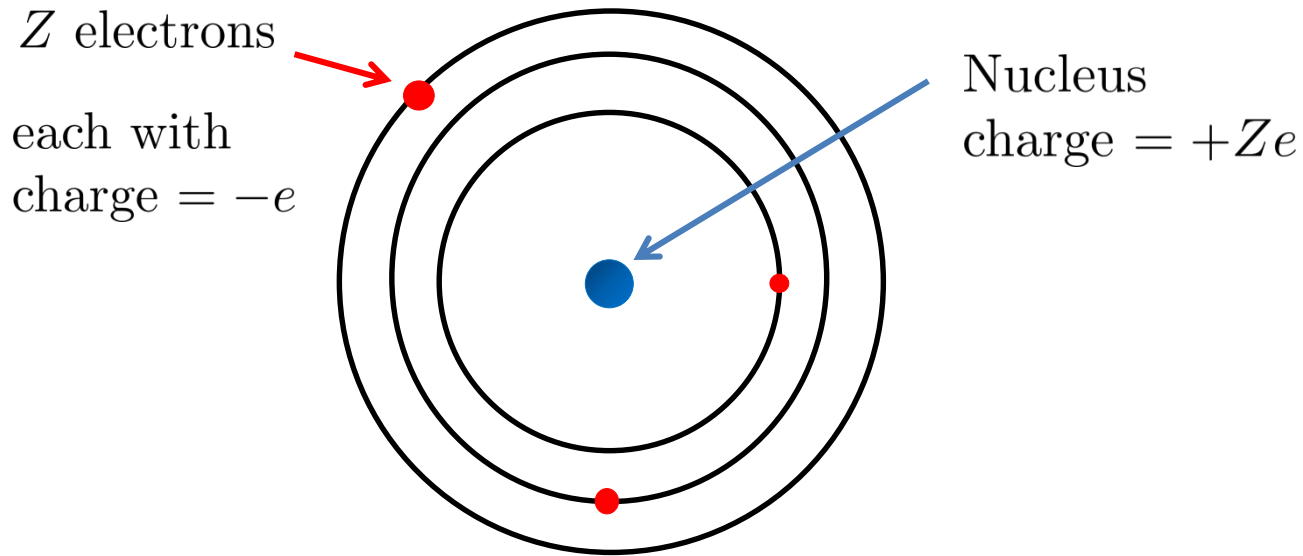
In **1905**, **Albert Einstein** extended Planck's idea of the quantum to explain the **photoelectric effect**. He proposed that the radiation itself is composed of discrete quanta with energy $E = hf$. These light quanta were later named photons, particles of light.



In **1910**, **Ernst Rutherford** was probing the **structure of the atom** with high energy alpha particles. He found that the positive charge and most of the mass is concentrated in a small volume – the atomic nucleus.

Rutherford's Model of the Nuclear Atom

Based on the conclusions of his scattering experiments which showed that the positive charge and most of the mass in an atom is concentrated in a small volume at the center, **Rutherford proposed a model of the atom that looks like a miniature solar system:**



But there's a big problem with this:

The electrons have uniform circular motion that requires continuous acceleration.

Accelerated charges emit electromagnetic waves (radiation) and lose energy.

The electron quickly spirals into the nucleus – i.e. **the atom is not stable!**

PhET 
H-atom

The Bohr Atom

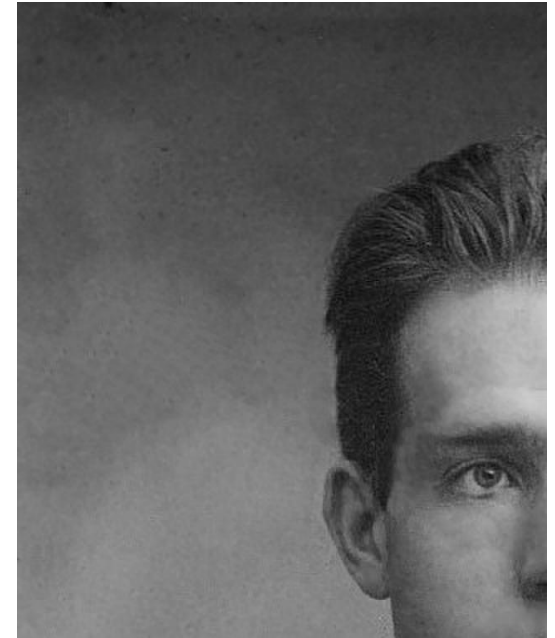
In 1913, a young Danish physicist, **Niels Bohr**, was working in Rutherford's lab. He wasn't having much success as an experimentalist, but in the last few months of his stay in England, he put together the **first working model of the atom**.

Bohr started with Rutherford's model and borrowed and extended Einstein's idea of the quantum to include not just the radiation, but the matter itself.

Bohr's starting point was three assumptions (Postulates):

1. The electron orbits the nucleus in one of a set of possible stable orbits of energy $E_n, n = 1, 2, 3 \dots$
2. The atom emits or absorbs a photon only when the electron changes orbits. The photon energy is
$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = |\Delta E_{\text{electron}}| \quad \text{i.e. energy lost or gained by the electron.}$$
3. The possible orbits are those with orbital angular momentum
$$L = n\hbar \quad \text{where : } \hbar = \frac{h}{2\pi} \quad n = 1, 2, 3, \dots$$

Bohr's work is amazing in its simplicity – a true back-of-the-envelope calculation; for the first time, the mysterious spectrum of Hydrogen could be understood.

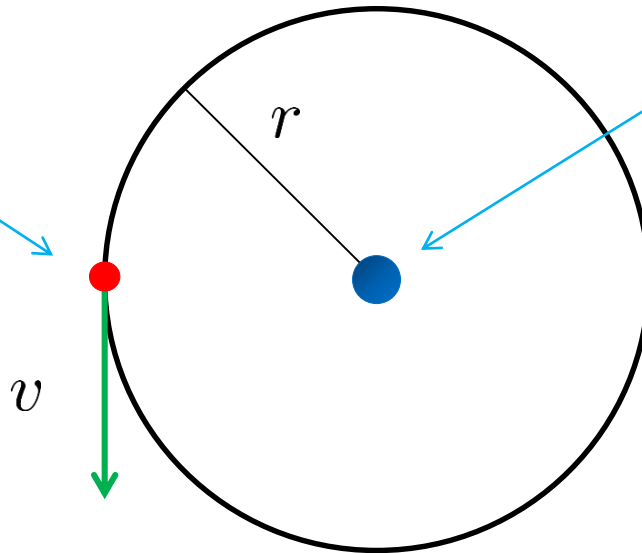


Whiteboard QM-5: Bohr Energy Levels of Hydrogen

Here, you will derive the energy levels of an electron in a Hydrogen atom the same way that Bohr did it in 1913. (Note: the derivation in your text uses de Broglie standing waves – more about these later today. This is not the way Bohr did it originally; he just used what he learned in PHY181 and PHY182.)

The Model:

Electron
charge = $-e$
mass = m



Nucleus (proton):
charge = $+e$
mass = m_p
 $m_p \gg m$
(to a good approximation,
the proton doesn't move)

Step 1: Use what you know about uniform circular motion and the electric force to find an expression for the square of the speed of the electron in terms of its radius and other physical constants. (LC)

$$\text{Answer: } v^2 = \frac{Ke^2}{mr}$$

Whiteboard QM-5: Bohr Energy Levels of Hydrogen

Step 2: Use Bohr's angular momentum postulate to obtain another expression for the speed of the electron. (LC) $\rightarrow (L = n\hbar, n = 1, 2, 3, \dots)$

$$\text{Answer: } v = \frac{n\hbar}{mr}$$

Step 3: Combine the expressions from steps 1 and 2 and find an expression for the allowed orbital radii of the electron. (LC)

$$\text{Answer: } r = \frac{n^2\hbar^2}{mKe^2} \quad \text{Or, } r_n = a_B n^2 \quad \text{where } a_B = \frac{\hbar^2}{mKe^2}$$

(The "Bohr Radius")

Step 4: Calculate the Bohr radius in nm (LC), and sketch the $n = 1, 2, \& 3$ orbits to scale. (look up the constants on our constants sheet; you should have them stored in your calculator)

$$\text{Answer: } a_B = 0.0529 \text{ nm} \quad 5$$


Whiteboard QM-5: Bohr Energy Levels of Hydrogen

Step 5: Write the total energy of the electron as the sum of its kinetic and potential energies. Use what you have from the previous steps to express the total energy, E , in terms of n and physical constants. (LC)

Answer: $E = -\frac{mK^2e^4}{2n^2\hbar^2}$ or, $E_n = -\frac{E_B}{n^2}$ where $E_B = \frac{mK^2e^4}{2\hbar^2}$
(The “Bohr Energy”)

Step 6: Calculate E_B the “Bohr Energy” (in eV) (LC); note, $-E_B$ is the ground state ($n = 1$) energy of the electron.

Answer: $E_B = 13.6 \text{ eV}$

Cass Bohr 

The Bohr Model of Hydrogen, a Summary

Allowed electron orbital radii
and Speeds

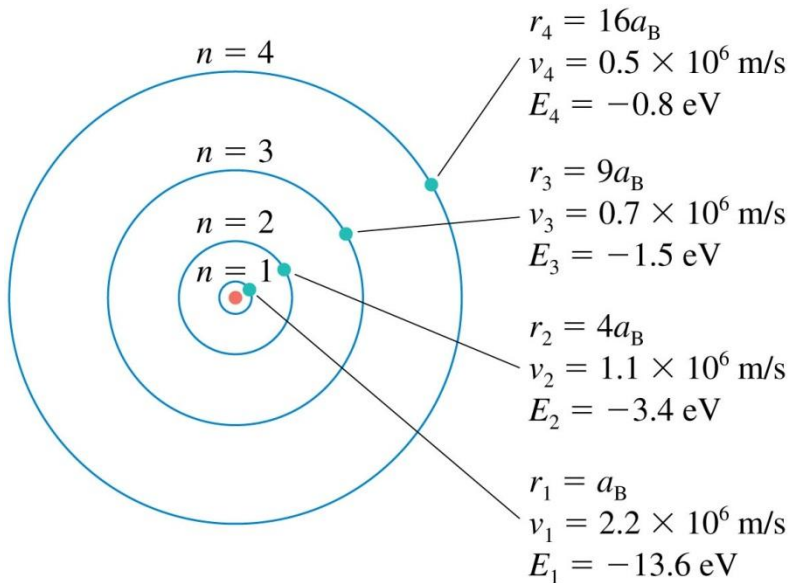
$$r_n = a_B n^2 \quad a_B = 0.0529 \text{ nm}$$

$$v_n = \frac{\hbar}{n m a_B} \quad n = 1, 2, 3, \dots$$

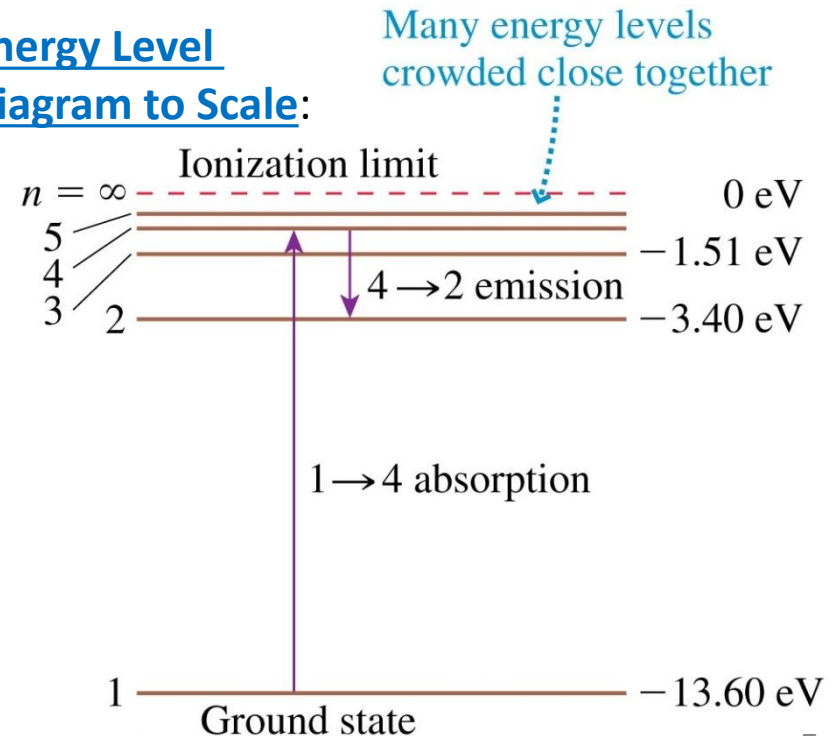
$$E_n = -\frac{E_B}{n^2} \quad E_B = 13.6 \text{ eV}$$

Allowed electron energy levels:

Orbit Sketch to Scale:

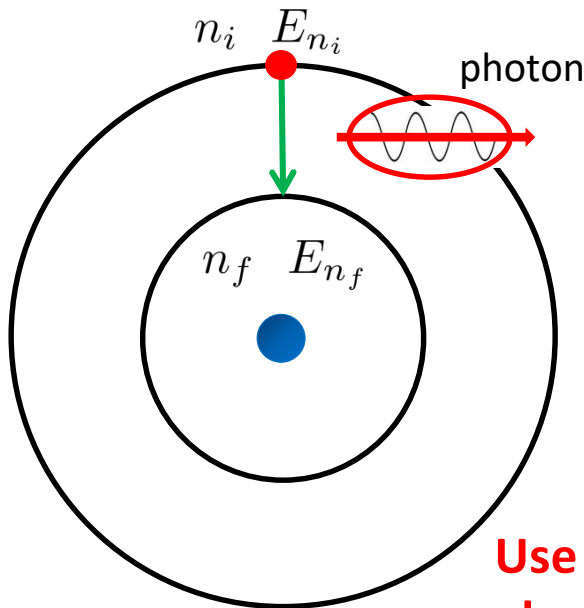


Energy Level
Diagram to Scale:

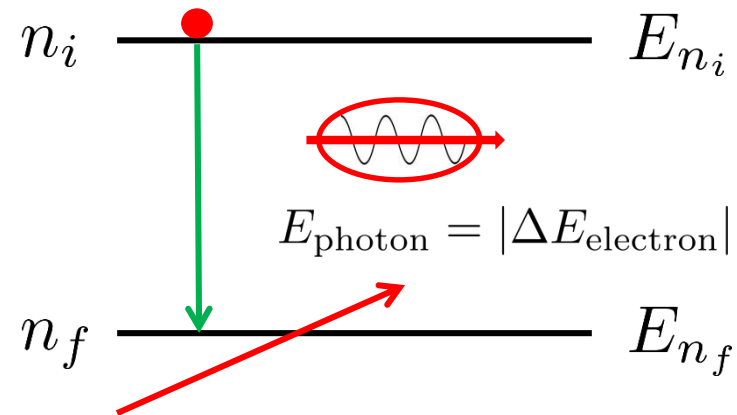


Whiteboard QM-6: What does Bohr's Model predict for the Hydrogen Spectrum?

Consider the emission of a photon:



Energy Level Diagram:



Use Bohr's postulate #2 and the allowed energies of the electron, $E_n = -13.6\text{eV}/n^2$, to find an expression for the wavelength of the emitted photon in nm in terms of the initial and final quantum numbers, n_i and n_f .

What is the wavelength for an $n = 3$ to 2 transition? (LC)

Answer: $\lambda = (91.18 \text{ nm}) \left(\frac{n_f^2 n_i^2}{n_i^2 - n_f^2} \right)$

Exactly the same as Balmer's formula.

PhET
H-atom 

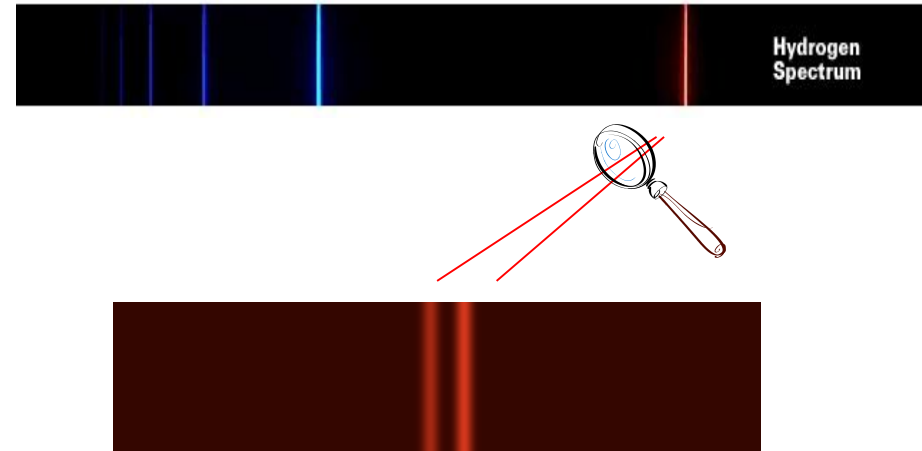
Problems with the Bohr Model

Bohr's model for the atom was a huge success in that it could finally explain the spectrum of Hydrogen to very high precision. **However, it left many important questions unanswered. For example:**

Multiple Electron Atoms: When applied to atoms with more than one electron, the model failed to reproduce spectra. Arnold Sommerfeld developed theories that included elliptical orbits, but these didn't work. *(Bohr's model does work for any one electron ion, like singly ionized Helium.)*

The Intensities of Spectral Lines:

Why are some emission lines brighter than others? This means that some electron transitions are more probable than others. Bohr says nothing about this. Also, some of these lines are actually closely spaced multiple lines – “fine structure”.



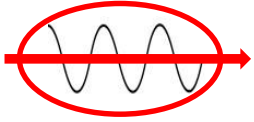
Postulates:

Where did the angular momentum postulate, “ $L = n\hbar$,” come from?

Ultimately, the Bohr Model gets some things right (the hydrogen energy levels) and gives a useful mental picture of the atom, but as a physical theory of the atom, it is incomplete. However, the postulate question leads us in the right path.

de Broglie and his Waves

Recall what we have for a photon:



speed, c

frequency, f

wavelength, λ

$$\text{Energy: } E = hf = \frac{hc}{\lambda}$$

$$\text{Momentum: } p = \frac{E}{c} = \frac{h}{\lambda}$$

(from special relativity*)

Note: (E & p) are particle properties; (f & λ) are wave properties.

In **1923**, [Louis de Broglie](#) (in his PhD thesis!) proposed that the above equations hold for all objects – **even particles have wave properties!**

What motivated de Broglie's thinking? No experiment had been done indicating that particles like electrons behave like waves.

He was more inspired by symmetry in nature:

“If photons have particle properties, shouldn't particles, like electrons, have wave properties?”



So, for every particle, de Broglie proposed that there is an associated wavelength:

$$\text{de Broglie Wavelength: } \lambda = \frac{h}{p}$$

**Don't use the equation $p = E/c$ for particles. The E in this equation must be the total relativistic energy which is hf for a photon, but is not the kinetic energy for particles. For particles, use wavelength = h/p .*

Whiteboard QM-7: de Broglie Wavelength

Part 1: What kind of numbers are we talking about? Consider a bullet:

$$m = 10 \text{ g}$$

$$v = 500 \text{ m/s}$$



Calculate the de Broglie wavelength of the bullet in meters. (LC)

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\text{Answer: } \lambda \sim 1.3 \times 10^{-34} \text{ m}$$

If we wanted to do an experiment to demonstrate the wave nature (e.g. diffraction or two-slit interference) of this bullet, what would we have to do?

We should pass the bullet through a hole about the same size as its wavelength! A tough experiment!

Part 2: The spacing between atoms in a crystal solid is on the order of 1 nm. **What kinetic energy (in eV) should an electron have so that its wave properties can be observed when passing through or reflecting from this crystal solid? (LC)**

$$\text{Solution: } \lambda = 1.0 \text{ nm} = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2} \frac{(mv)^2}{m} = \frac{p^2}{2m}$$

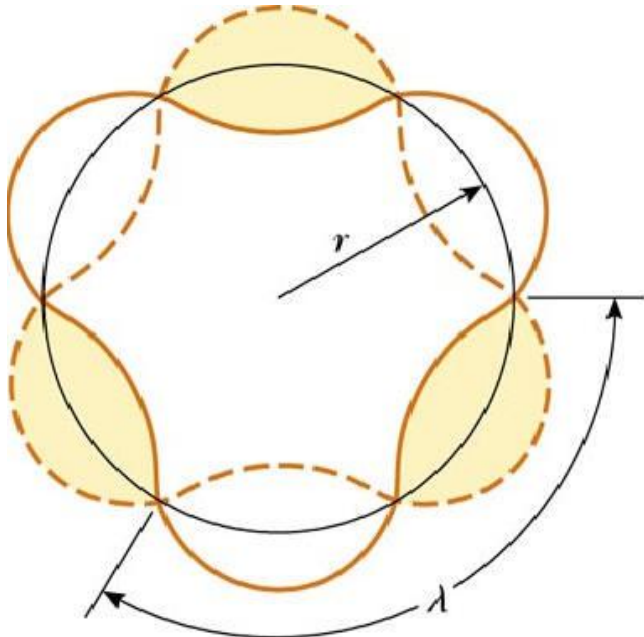
$$\text{So: } K = \frac{h^2}{2m\lambda^2} = 2.41 \times 10^{-19} \text{ J} = 1.51 \text{ eV}$$

It is very easy to produce electrons in the laboratory with kinetic energies of a few eV. The wave nature of electrons was demonstrated experimentally (by accident) by Davisson and Germer in 1927.

How did de Broglie know that he was on to something really big?

De Broglie realized this connection between his wavelength and the Bohr theory of hydrogen:

“The allowed electron orbits in the hydrogen atom are those orbits for which the circumference contain an integral number of de Broglie wavelengths, i.e. circular standing waves.”



For any Bohr orbit (i.e. any n), de Broglie says:

$$2\pi r_n = n\lambda = n\frac{h}{p}$$

or, $pr_n = n\frac{h}{2\pi}$

and, $pr_n = \underline{L} = n\hbar$

(Bohr's angular momentum postulate!)

PhET
H-atom 

But, is an electron a particle or a wave? And, if it has wave properties, what is doing the waving?

[Cass qm debr](#) 

By the way: where was A.H. Compton from? Wooster, OH!

Whiteboard Problem: QM-8 – Now a HW problem

(a simple, but reasonable model of the nucleus)

The diameter of the atomic nucleus is about 10 fm. A simple model of the nucleus is that protons and neutrons are confined in a one dimensional box of length $L = 10$ fm. **Consider a proton in such a box:**

a) **What are the lowest two energy levels of the proton (in MeV)?**

Enter the lowest energy (ground state) in LC.

Hint: Find the two longest de Broglie wavelengths of the proton that can form **standing waves** in the box of length, $L = 10$ fm. **Remember this from standing waves: the wave must have nodes at the ends.** Then use these wavelengths to find the kinetic energies.

b) **What is the wavelength (in nm) of a photon that is emitted by a nucleus when the proton makes a transition from the $n = 2$ to the $n = 1$ state ? (LC)**

What type of electromagnetic radiation is this?