QM-1: Introduction to Quantum Mechanics

For the rest of the semester, we're going to cover an introduction to quantum mechanics.

Why are we doing this?

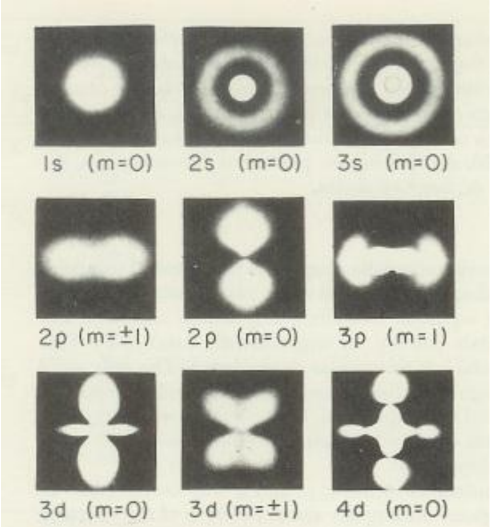
Reason #1: Physics is a Liberal Art . . . a significant accomplishment of our species

- For many of you, PHY181/182 are the only physics courses that you will take.
- In the 21st century, do physicists spend their time sliding blocks down inclines?
- No, these problems are already solved.
- You, as a person educated in the liberal art of physics, should have some understanding of what has been going on in physics in the last 100+ years.

Reason #2: Quantum Mechanics is Useful

- Many of you are chemistry majors, "*atomic and molecular engineers*".
- Atoms can only be explained using quantum mechanics. **What are these funny things?** →
- For you engineering majors: the entire information industry is based on the semiconductor which can only be explained with quantum mechanics.

From my chemistry book (c. 1970's):



Reason #3: Have you heard of Quantum Computing?

Reason #4: Quantum Mechanics is Fun!

The History of Quantum Mechanics

The two great revolutionary theories of 20th century physics, Relativity and Quantum Mechanics, have very different histories.

Relativity (both Special and General) was developed by one man, Albert Einstein. (*We'll look at Relativity in PHY182*)

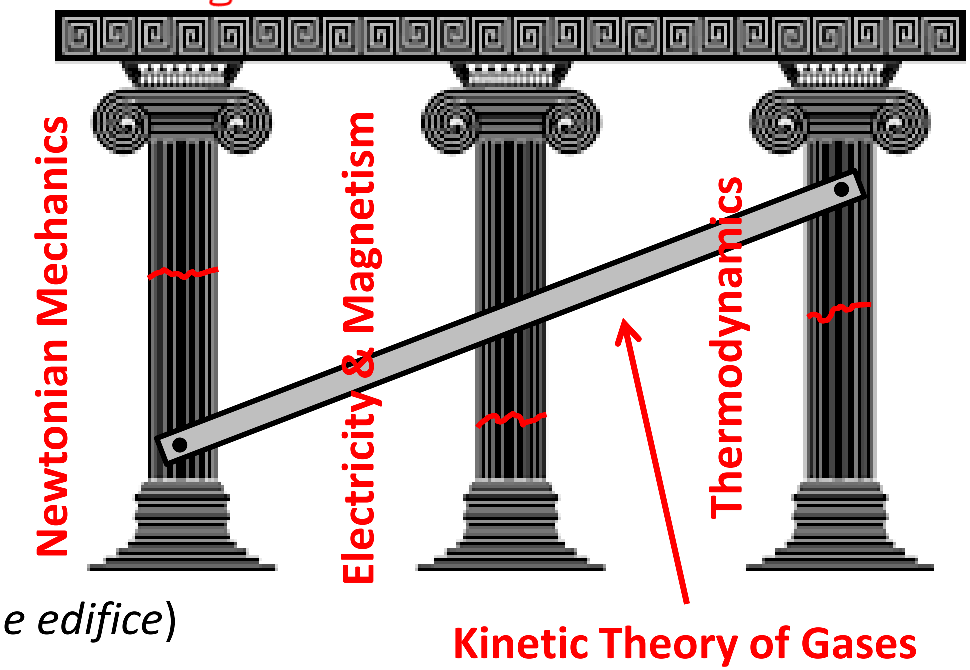
Quantum Mechanics was developed by many people over a period of about 25 years.

The development was very painful – the physics world was dragged kicking and screaming into accepting quantum mechanics!

Nobody likes quantum mechanics, but it works! ***(in fact, it works really really well!)***

Let's go back to the beginning –
The State of Physics in 1900.

Physics in 1900



The State of Physics in 1900 was Excellent!

"Just about everything that needed to be explained had already been explained!"

Some quotes from 19th century Physicists.

Why could they say this?

But, there were some seemingly insignificant questions: (*small cracks in the edifice*)

What about:

- The electric field of a moving charge?
- The Michelson Morley experiment and the Ether?
- The electromagnetic spectrum of a blackbody?
- The photo electric effect?
- The spectrum of Hydrogen?
- Radioactive decay?

The first two questions → **Special Relativity** (and then General Relativity)

The blackbody spectrum and the photo electric effect → **Quantum Theory of Light**

The spectrum of Hydrogen → **Quantum Theory of Matter**

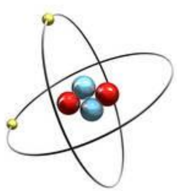
Radioactive decay → **Nuclear Structure & Forces**

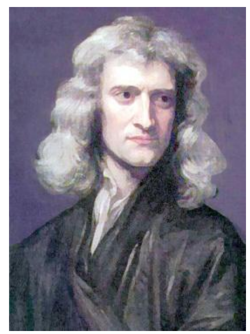
The cracks represented by these questions would grow and bring the whole structure down, and new physics would be built that would contain the old, but also explain the new discoveries.

Did Quantum Mechanics Prove Newton Wrong?

Not really, both Relativity and Quantum Mechanics put bounds on Newtonian Physics that define where it is an accurate description of reality, but does not apply beyond those boundaries.

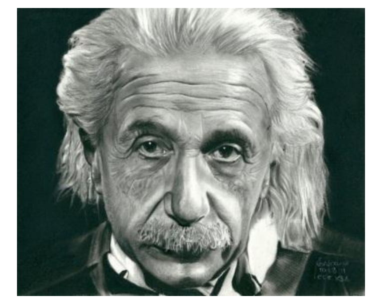
Quantum Mechanics





Newtonian Physics

Special and General Relativity

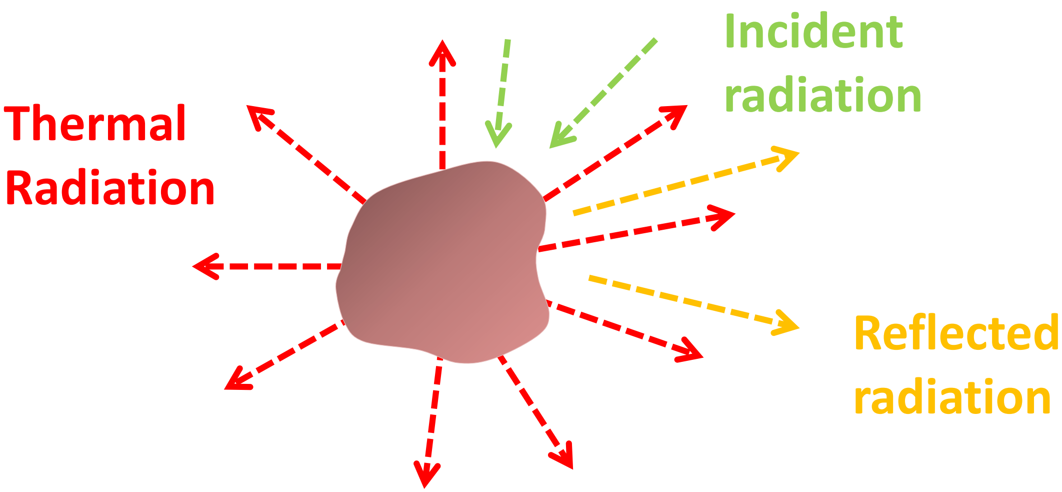


Someday, someone will discover the bounds for both Relativity and Quantum Mechanics!

QM all started with - - The Spectrum of Blackbody Radiation

Thermal Radiation:

Any object, not at zero temperature, emits electromagnetic radiation called thermal radiation. When we measure the radiation intensity of a real object, what do we measure?



Measures light intensity

Measures both thermal radiation and reflected radiation

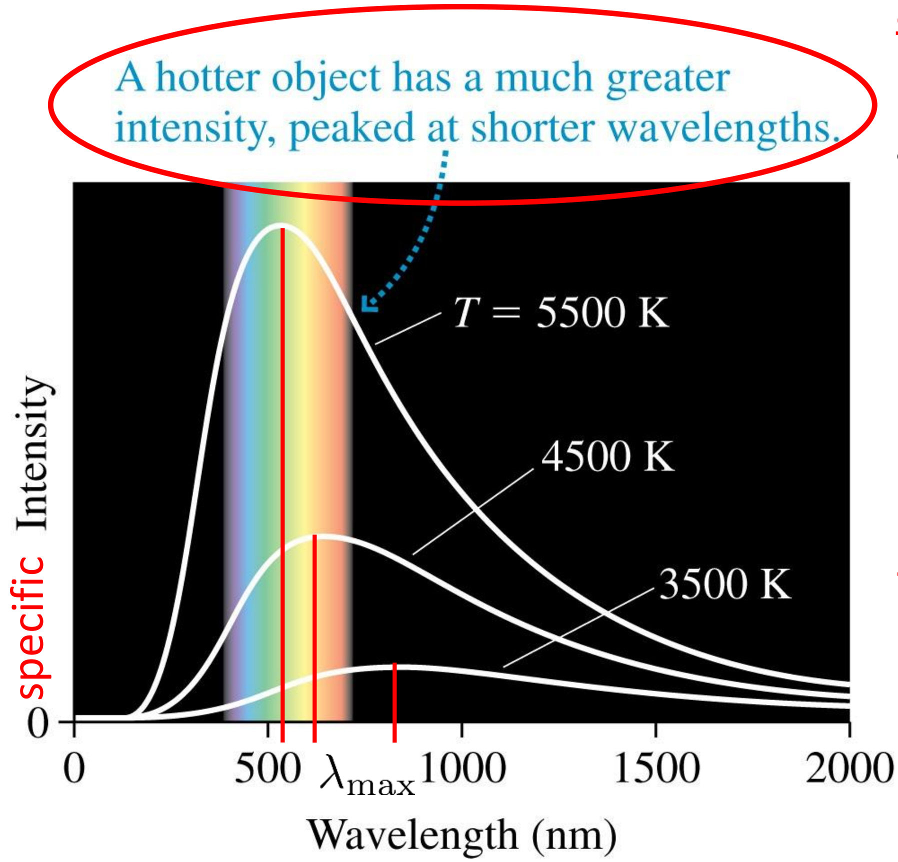
A **blackbody** **is a perfect thermal emitter**. It absorbs all incident radiation, so that the radiation measured from a blackbody is just its own thermal radiation.

Why were scientists in the 1890's so interested in thermal radiation?

Because of the recently invented incandescent light bulb - a case of thermal emission. They wanted to understand it to build better light bulbs.

The Spectrum of Blackbody Radiation - Observations

Measuring the specific intensity, I_{sp}, of radiation of a blackbody at different wavelengths and temperatures produces plots like these:



Note: λ_{max} is the wavelength of maximum intensity, not a maximum wavelength.

Stefan-Boltzmann Law: The intensity for a higher temperature is everywhere greater than for a lower temperature.

$$I = \int_0^\infty I_{sp}(\lambda, T)\,d\lambda = \sigma T^4$$

$$\sigma = 5.67 \times 10^{-8} \frac{W}{m^2\ K^4}$$

(Stefan-Boltzmann constant)

Wien Displacement Law: The wavelength of maximum intensity decreases at higher temperatures.

$$\lambda_{max} T = \text{constant} = 2.90 \times 10^6\ nm\ K$$

PhET BB

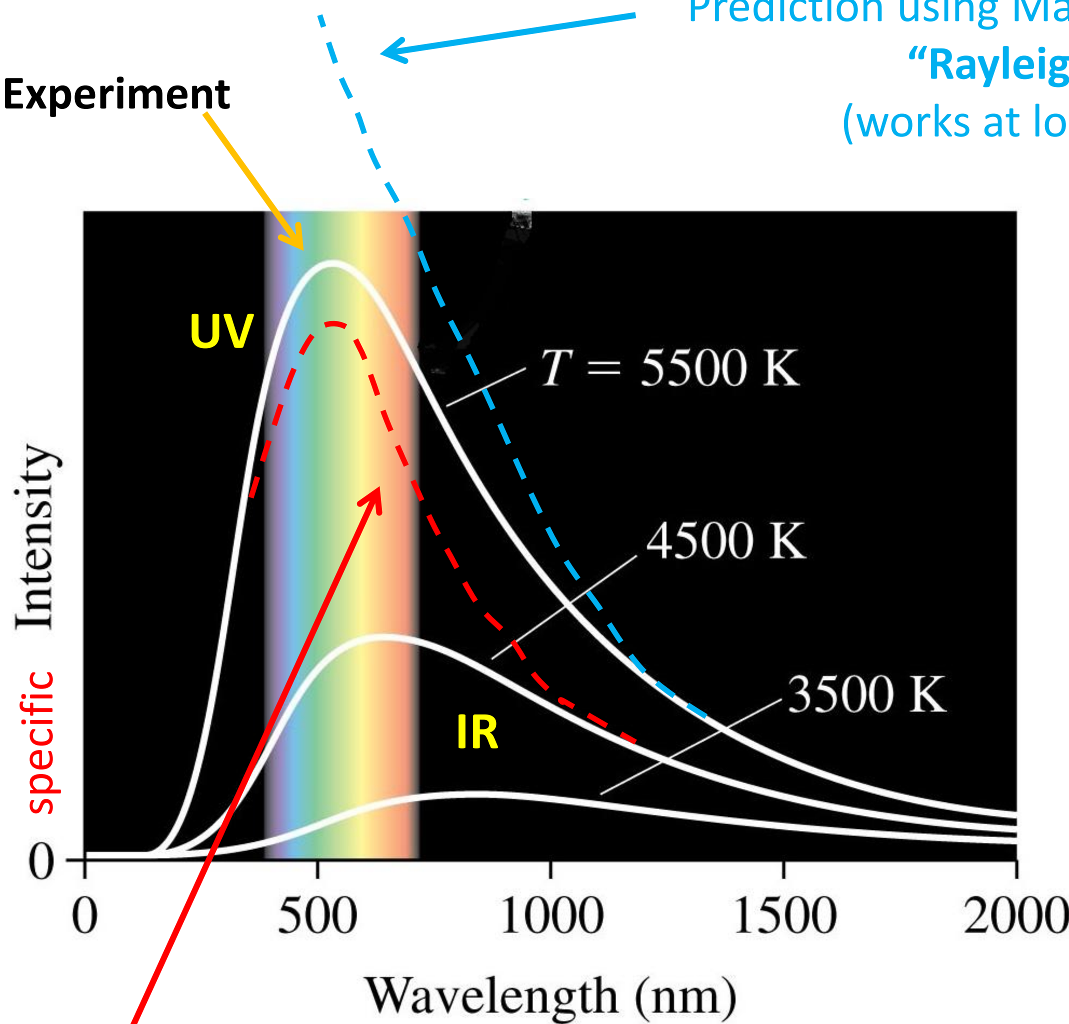
Whiteboard Problem QM-1

A 2.0 cm diameter metal sphere is glowing red, but a spectrum shows its emission spectrum peaks at an infrared wavelength of 2.0 micrometers.

How much power does the sphere radiate? (LC)

The Spectrum of Blackbody Radiation - Theory

Prediction using Maxwell's Electromagnetism:
"Rayleigh Jeans Law"
(works at long wavelengths)



Wien's Curve Fit
(works at short wavelengths)

In 1900, **Max Planck** fit this mathematical function to the experimental curve:

$$I_{\mathrm{sp}}(\lambda, T) = \frac{A}{\lambda^5}\left[\frac{1}{e^{\frac{B}{\lambda k T}} - 1}\right]$$

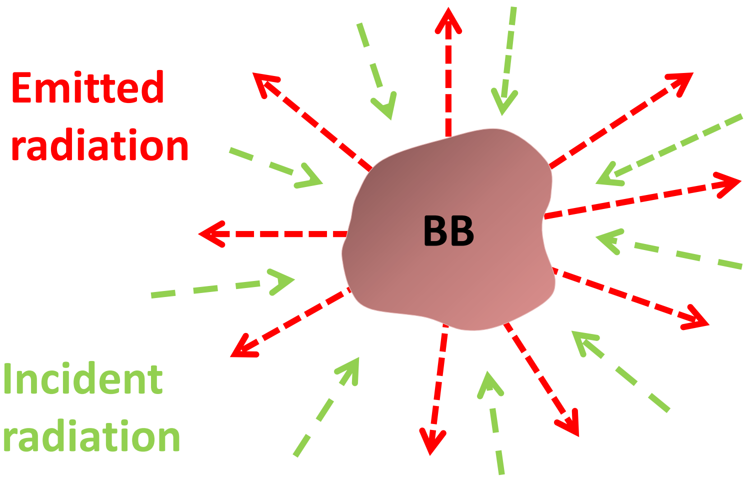
Where k is Boltzmann's constant from thermodynamics, and A and B are unknown adjustable constants.

Planck found that with proper choices of A and B, this function fit the observed spectra exactly.

So there must be some physics behind it!

Planck's Hypothesis

Planck found that he could derive his curve fit from physics only if he included a very **unusual and remarkable assumption** about how the blackbody emitted and absorbed radiation:



The BB can absorb or emit radiation only in discrete chunks or "quanta" with energy:

$$E = nhf$$

Where: $f = \text{frequency}$

$n = 1, 2, 3, \cdots$

$$h = \text{Planck's Constant} = 6.626 \times 10^{-34} J \cdot s$$

With this assumption, Planck could show that the specific intensity of a BB is:

$$I_{\rm sp}(\lambda, T) = \frac{2\pi h c^2}{\lambda^5}\left[\frac{1}{e^{\frac{hc}{\lambda k T}} - 1}\right] \qquad (c = \text{speed of light})$$

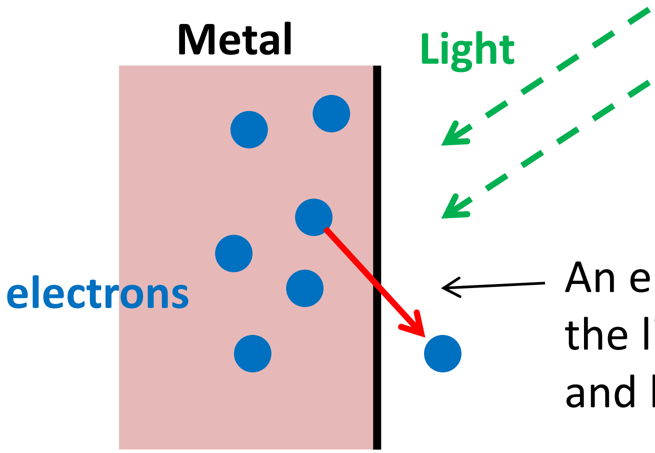
But: the assumption, *E=nhf*, violates Maxwell's theory which permits radiation to be emitted and absorbed over a continuum of energies.

The "chunkiness" is contained in the constant h. Planck wanted let h go to zero after the derivation, but that gives the Rayleigh Jeans Law, so h was here to stay.

Cass BB

Photoelectric Effect

In 1886, Heinrich Hertz discovered (by accident) that light shining on a metal can release electrons:

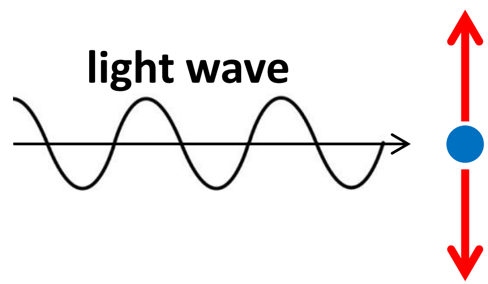


frequency, f
wavelength, λ
intensity, I

PhET PEE
with waves

An electron can absorb energy from the light escaping from the metal and be observed.

Classical Explanation:



Wave oscillates the electron back and forth until the electron has enough energy to escape the metal.

So the electron energy should depend on the light intensity (square of wave amplitude). Light of any frequency, given enough time, should free electrons.

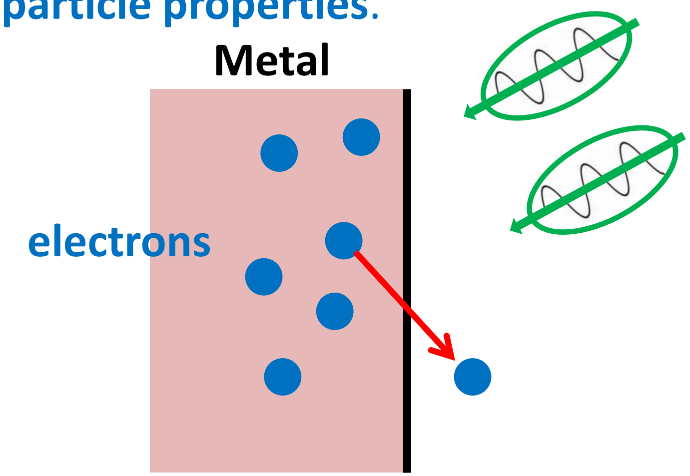
But, experiments showed that:

If: the light frequency, $f < f_0$, a threshold frequency, no electrons are freed, regardless of the intensity

PhET PEE
with waves

Photoelectric Effect – Einstein's Explanation

In 1905, Albert Einstein borrowed and extended Planck's idea of the quantum to explain the photoelectric effect. **He proposed that Planck's quantization rule should apply to the radiation itself, i.e. light is composed of discrete chunks or quanta that also have particle properties**.



Light Particles: "Photons"

$$\text{photon energy, } E = hf = \frac{hc}{\lambda}$$

f = light frequency
λ = light wavelength
h = Planck's constant
c = speed of light

According to Einstein's explanation:

There is some minimum energy, E_0, for an electron to escape

If a photon with energy, $E = hf > E_0$ is absorbed by the electron, the electron escapes the metal, regardless of intensity.

If none of the photons have $E = hf > E_0$, no electrons escape.

So the photoelectric effect is an interaction between a single photon and a single electron.

PhET PEE (with photons)

Cass PEE

Whiteboard Problem QM-2

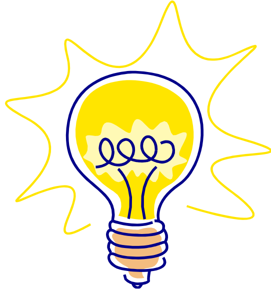
a) **Determine the energy, in eV, of a photon with a 550 nm wavelength. (LC)**
b) **Determine the wavelength of a 7.5 keV x-ray photon. (LC)**

This problem introduces us to a new unit of energy, **The Electron-Volt (eV),** which is very useful on the atomic scale. It is defined as the energy acquired by an electron accelerated through a potential difference of 1 Volt – we'll learn about that in PHY182.

For now, all we have to know is the conversion factor between electron-volts and Joules:

$$1.0\ eV = 1.6 \times 10^{-19}\ J$$

We will see that the typical energies of electrons in an atom are a few eV's.



Whiteboard Problem QM-3

A 100 W incandescent lightbulb emits about 5 W of visible light. (The other 95 W are emitted as infrared radiation or lost as heat to the surroundings.) The average wavelength of the visible light is about 600 nm, so make the simplifying assumption that all the visible light has this wavelength.

How many visible light photons does the light bulb emit per second? (LC)

Hint: Look at the units:

$$\text{Photon energy, } E = hf = \frac{hc}{\lambda} \text{ (energy/photon)}$$

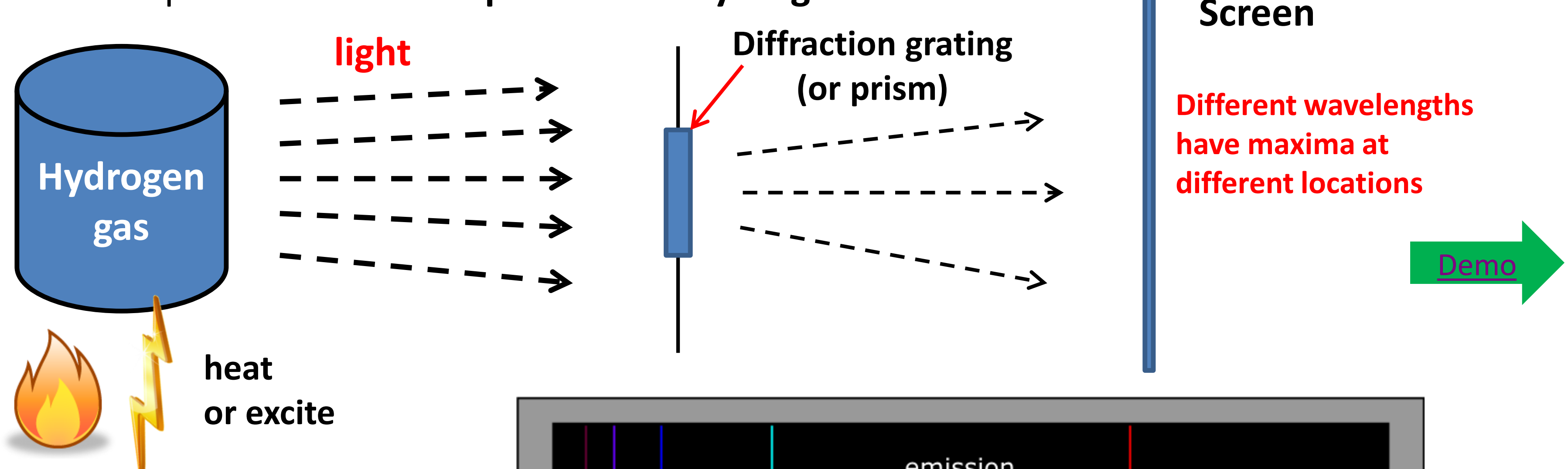
$$\text{Light Power, } P, \ (W = J/s)$$

The Atom

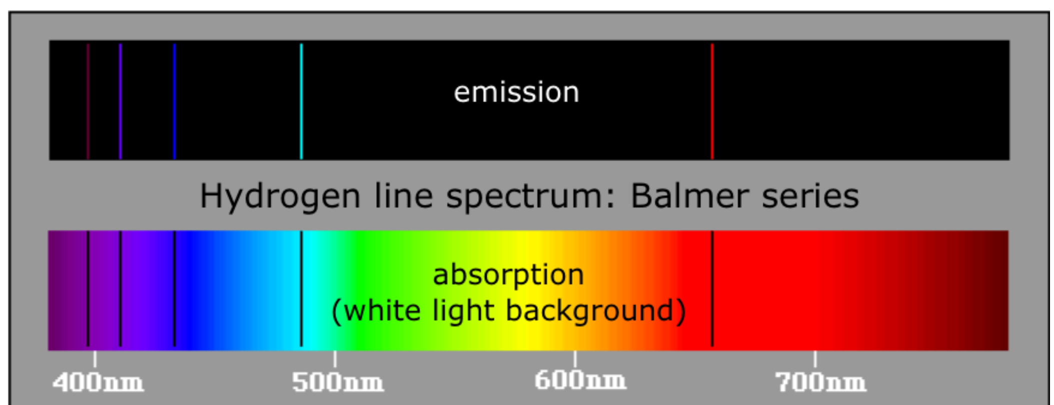
What was known about the atom in 1900? First, the existence of atoms was not universally accepted at this time, but for those who did think atoms existed, they knew:

1. **Atoms are small, but they are stable.**
2. **Atoms are electrically neutral, but contain negative electrons.**
3. **Atoms emit and absorb electromagnetic radiation (i.e. light)**

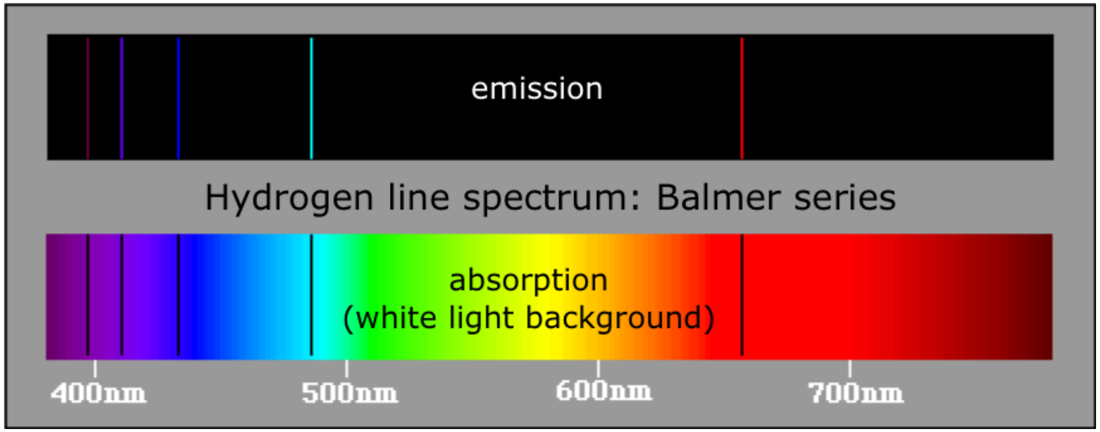
For example: **the emission spectrum of Hydrogen:**



Result for Hydrogen



Location of Hydrogen Spectral Lines



Try plugging in the numbers to get these lines

In 1884, a Swiss school teacher, **Johann Balmer**, found a formula by trial and error that reproduces the locations of the Hydrogen spectral lines:

$$\lambda = (91.18\ nm)\left(\frac{n_0^2 n^2}{n^2 - n_0^2}\right) \qquad \text{where: } n_0 = \text{integer} \neq 0, \quad n = \text{integer} > n_0$$

$$n_0 = 1 \Rightarrow \text{Lyman Series in the Ultraviolet}$$
$$n_0 = 2 \Rightarrow \text{Balmer Series in the Visible}$$
$$n_0 = 3 \Rightarrow \text{Paschen Series in the Infrared}$$

The Problem of the Atom

Possibly the central question (***the Holy Grail***) of physics in the early 20th century was this:

What internal structure of the atom would yield the observed patterns of spectral lines?

The efforts to answer this question would ultimately lead to a new physics,

Quantum Mechanics

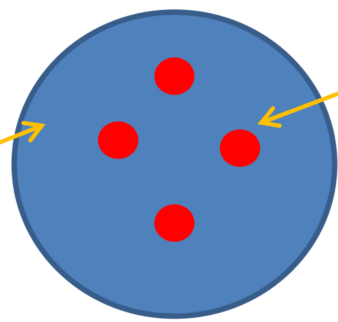
The Thomson "Plum Pudding" Model . . . a first try: *(Thomson discovered the electron in 1897)*

"Pudding"

Uniformly Charged Positive sphere

$\text{charge} = +Ze$

$(Z = \text{atomic number})$



Atom

"Plums"

Z negative electrons each with charge $-e$

$$e = \text{electronic charge}$$
$$= 1.6 \times 10^{-19} \text{ Coulombs}$$

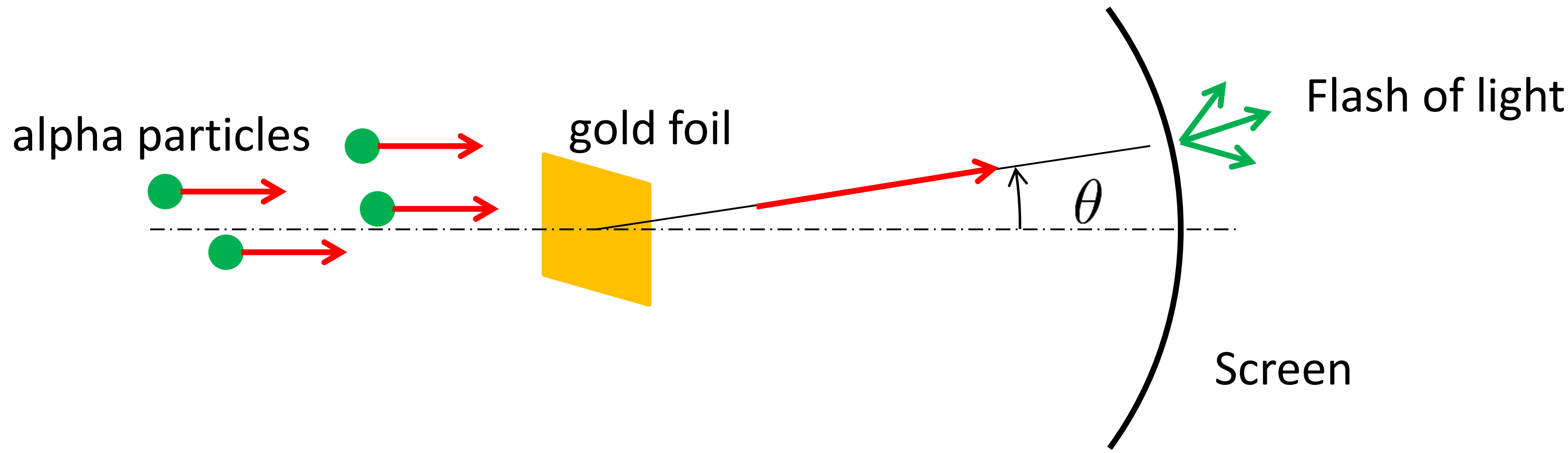
The electrons vibrate back and forth, thus emitting and absorbing radiation.

However, the calculated spectra aren't anywhere close to the observations.

PhET H-Atom

Rutherford Scattering

In 1910, Ernst Rutherford (and his assistants Geiger and Marsden) fired high energy alpha particles (Helium nuclei) at a thin sheet of gold foil. They didn't expect much to happen.



If the gold atoms were like Thomson's plum pudding model, there would be very little chance of the alpha particles scattering at high angles.

PhET Ruth Scat

So Rutherford was quite surprised when Geiger and Marsden reported that they had observed alpha particles scatter at high angles – up to 180º! Rutherford's reaction:

"It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."

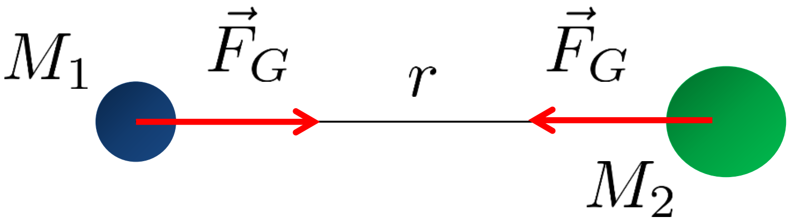
The Conclusion: **This can only happen if the positive charge in an atom is concentrated in a small volume – The Nucleus.**

PhET Ruth Scat

The Basics of the Electric Force

(we'll do a lot more with this in PHY182; if you've seen it before, help your other group members)

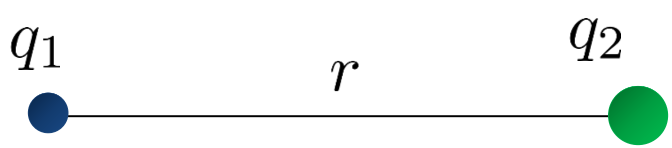
Recall what Newton said for the **Gravitational Force between two masses**:



$$\vec{F}_G = \left(\frac{GM_1M_2}{r^2}, \text{ attractive} \right)$$

$$U_G = -\frac{GM_1M_2}{r}$$

In 1785, **Charles Coulomb** proposed something similar for the **Electric Force between two electric charges, q_1 and q_2**, where charge is measured in Coulombs (C):



The Electric Force: (Coulomb's Law)

$$\vec{F}_E = \left(\frac{K|q_1||q_2|}{r^2}, \begin{array}{c} \text{repulsive for like charges} \\ \text{attractive for unlike charges} \end{array} \right)$$

The Electric Potential Energy:

$$U_E = \frac{Kq_1q_2}{r}$$

$$K = \text{Coulomb constant} = 8.99 \times 10^9 \ \frac{Nm^2}{C^2}$$

Note for the electric potential energy, the sign is determined by the sign of the charges.

Whiteboard Problem QM-4: Rutherford Scattering

An alpha particle with kinetic energy 8 MeV rebounds at 180^o from a Gold Nucleus.

What is the alpha's closest approach distance to the nucleus? (LC)

Hint: Use the electric potential energy and energy conservation. You may assume the mass of the nucleus is much much larger than the mass of the alpha, so the nucleus does not recoil.

The alpha particle is composed of 2 protons and 2 neutrons and has a mass ~ 4 m_p and a charge = +2e. The Gold nucleus has a charge of +79e.
(see our constants sheet for values of the proton mass, m_p, the electronic charge, e, and the Coulomb constant, K)

This closest approach distance puts an upper limit on the size of the nucleus in the Gold atom.