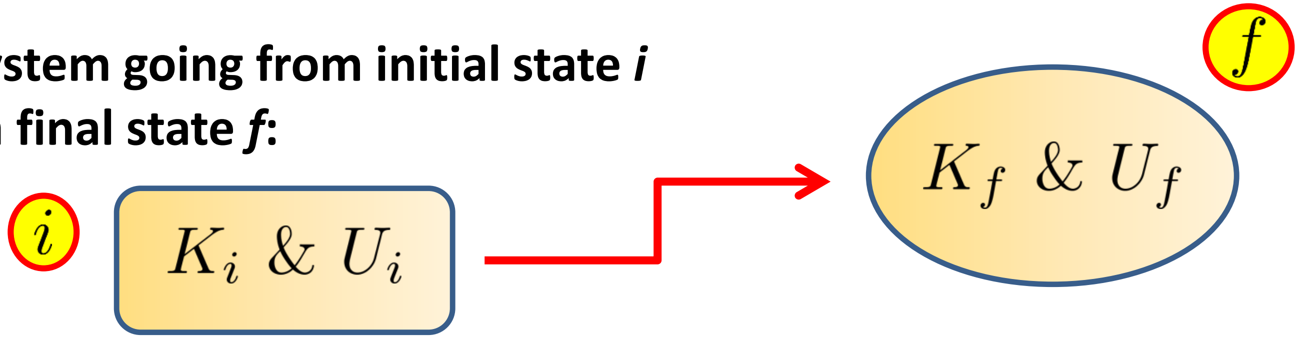
9/10-3: Where We Stand with Conservation of Energy

For a system going from initial state *i* to a final state *f*:



If there are no dissipative forces (e.g. friction) or applied forces, the Mechanical Energy is Conserved.

$$E_{mech} = K + U = \text{constant}$$

$$\Delta E_{mech} = \Delta K + \Delta U = 0$$

Where: $\Delta \overset{\text{(always)}}{=} \text{(final) - (initial)}$

Where:

$$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

For Uniform Gravity:

$$\Delta U_g = mgy_f - mgy_i = mg\left(y_f - y_i\right)$$

For a Linear Spring:
(with x_s = 0 at equilibrium)

$$\Delta U_s = \frac{1}{2}kx_{s_f}^2 - \frac{1}{2}kx_{s_i}^2 = \frac{1}{2}k\left(x_{s_f}^2 - x_{s_i}^2\right)$$

What about other forces that can't be described with a Potential Energy?

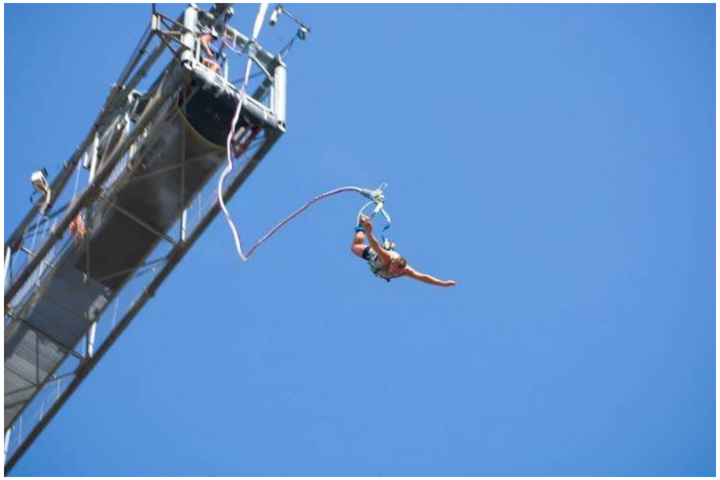
But before we do that, let's do a really fun problem.

A fun start to the day: Whiteboard Problem 9/10 – 10

In a moment of insanity, you decide to go bungee jumping. You stand on a bridge 100 m above a raging river and attach a 30 m long bungee cord to your harness. A bungee cord, for practical purposes, is just a long spring, and this one has a spring constant of 40 N/m. Assume that your mass is 80 kg. After a long hesitation, you dive off the bridge.

How far are you above the water when the cord reaches its maximum elongation and you instantaneously come to rest?

(LC – a 2 point shot)



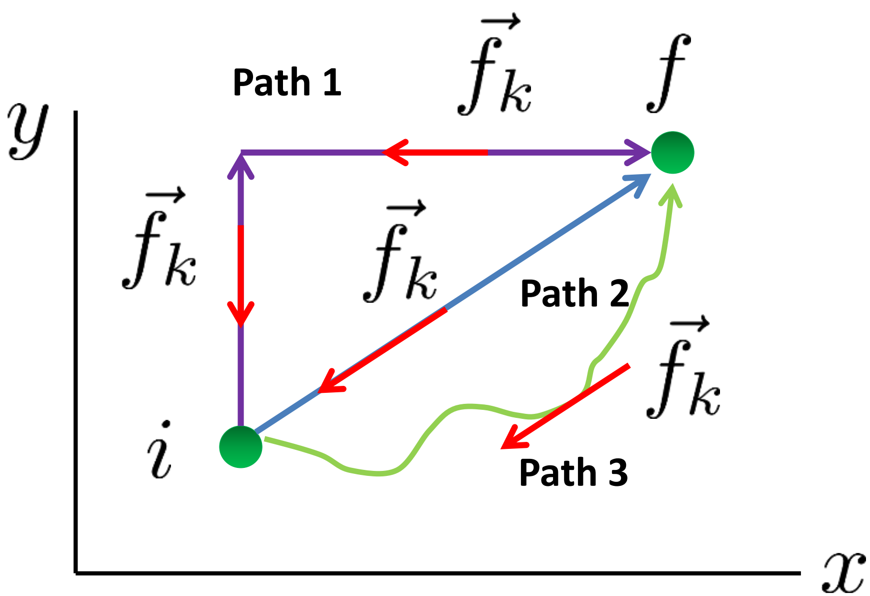
Here's what's going on in Bungee Jumping.

This is a fairly easy problem – If you just Follow the Procedure

Nonconservative* Forces

Definition: A **nonconservative force** is a force for which the work done by the force in going from some initial point *i* to some final point *f* **does depend on the path followed.**

For Example, Friction:



The work done by the friction force is different for each path from *i* to *f*.

So, we cannot associate a potential energy with a nonconservative force.

But, can we include nonconservative forces in our conservation of energy technique?

. . . Yes!

**Yes!*

**Again, as we'll see, the name comes from the fact that nonconservative forces do not conserve the mechanical energy.*

The Complete Conservation of Mechanical Energy

We started with the **Work – Kinetic Energy Theorem** for a system going from an *initial state, i*, to a *final state, f:*

$$\Delta K = W_{\text{net}}(i \to f) = W_c(i \to f) + W_{nc}(i \to f)$$

conservative forces (→ W_c) nonconservative forces (→ W_{nc})

$$\text{For Conservative Forces*: } \Delta U = -W_c(i \to f)$$

$$\text{Define: Total Mechanical Energy: } E_{\text{mech}} = K + U$$

$$\text{so, } \Delta E_{\text{mech}} = \Delta K + \Delta U$$

So, the Work – Kinetic Energy Theorem can be written as:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}$$

This is the equation we've been using with the added feature that **we now know how to include forces like friction or applied forces**. ***They go right here.***

We see that potential energy is energy that can be stored and returned to kinetic. Applied forces can change the kinetic or the potential energies, and hence the mechanical energy. On the other hand, dissipative forces, like friction, will always decrease the mechanical energy and increase the thermal energy.

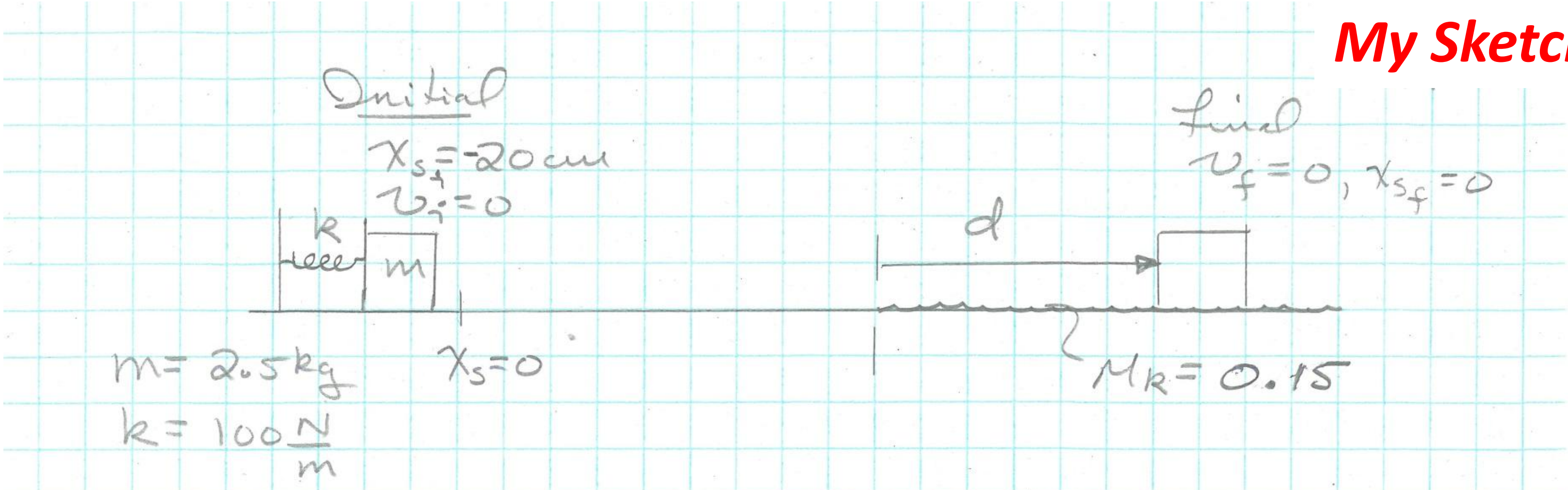
*If you have more than one conservative force, you'll need a potential energy for each.

Whiteboard Problem: 9/10-11

A horizontal spring with spring constant 100 N/m is compressed 20 cm and used to launch a 2.5 kg box across a frictionless horizontal surface. After the box travels some distance, the surface becomes rough. The coefficient of kinetic friction between the box and the surface is 0.15.

How far does the box slide across the rough surface before it comes to rest?* **(LC)**

My Sketch



If you think about it, since the spring force is not constant, doing this problem using dynamics and kinematics would be rather tough.

Whiteboard Problem: 9/10-12

Kathy's baby brother sits on a sled. The combined mass of the sled and the baby is m. Kathy pulls on the rope that is at an angle θ above the horizontal. The tension T is constant and the coefficient of kinetic friction between the sled and the snow is μ_k.

Find an expression (*in terms of the given quantities, constants, and numbers*) for the speed of the sled after Kathy has pulled it a distance d starting from rest. **(LC, a 2-point shot!)**

***Careful*: there are two traps in this problem - - don't fall into them!**

Force from Potential Energy

We know how to find the Work done by a Force:

$$W = \int \vec{F} \cdot d\vec{s}$$

By the way:
This should be your answer if you are ever asked "What is Work" – not F x d!

Or, in one dimension:

$$W = \int F_x dx$$

And for a conservative force:

$$\Delta U = -W(\text{done by } \mathrm{F_x}) = -\int F_x dx$$

$i.e.$ if we know F_x, we can find ΔU.

Can we go the other way? i.e. if we know the potential energy, can we find the force? As your author shows (for one dimension):

$$F_x = -\frac{dU}{dx} = -(\text{slope of the } U \text{ vs. } x \text{ curve})$$

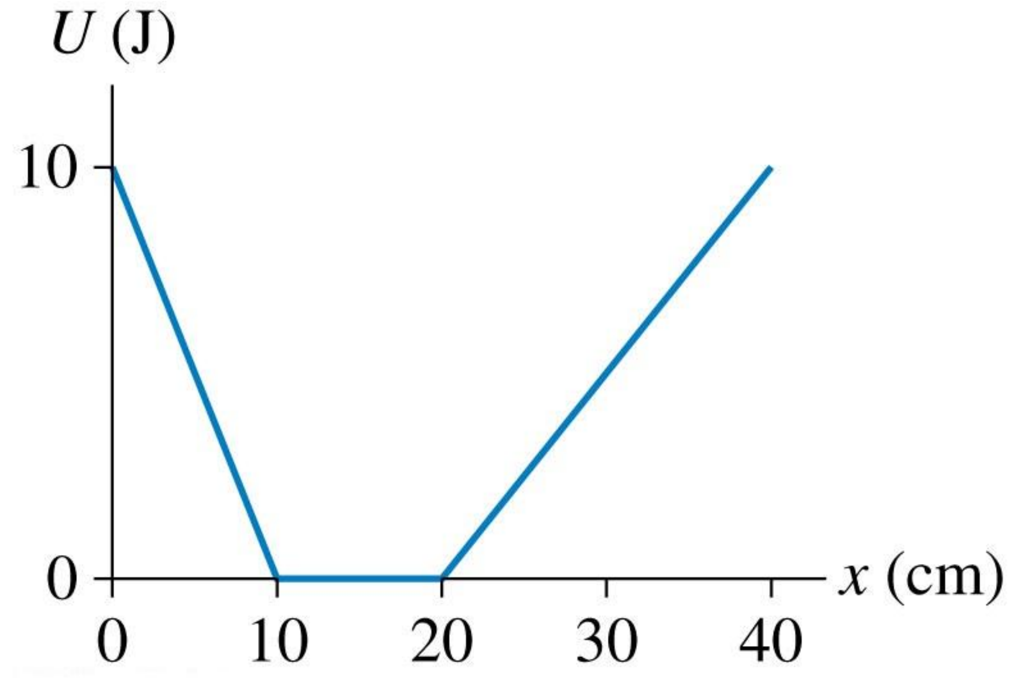
Everyone forgets this negative sign, but it's important; you just have to remember it, or remember that particles always go downhill.

So, a changing potential energy implies a force is present.

Whiteboard Problem 9/10 – 13

A particle has the potential energy show below.

On LC: **Sketch a graph of x-component of the force, F_x, vs. x.**



Whiteboard Problem 9/14

A particle moving along the x-axis has the potential energy:

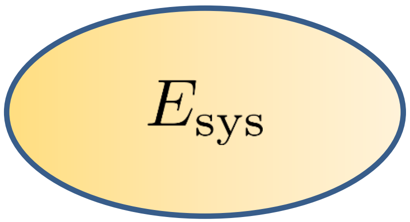
$$U(x) = \frac{10}{x}\ J$$

where x is in m.

What is the x-component of the force on the particle at x = 5 m? (LC)

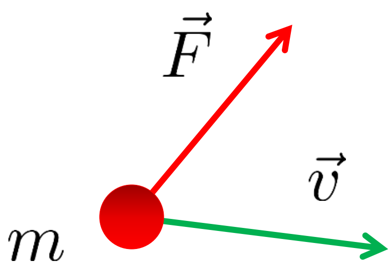
Power

For a system, **Power is the rate that energy is transferred into or out of the system:**



$$\text{Power, P} = \frac{dE_{\text{sys}}}{dt} \qquad \text{Units: } 1\ \tfrac{J}{s} = 1\ \text{Watt}(W)$$

For a mass for which the energy is changing because a force is doing work on it, your author shows:



$$\text{Power, P} = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$$

Whiteboard Problem 9/10 – 15

You push a 10 kg block of steel across a steel table at a steady speed of 1.0 m/s for 3.0 s.

How much work did you do, and what was your power output (LC) while pushing?

For steel on steel: $\mu_k = 0.6$