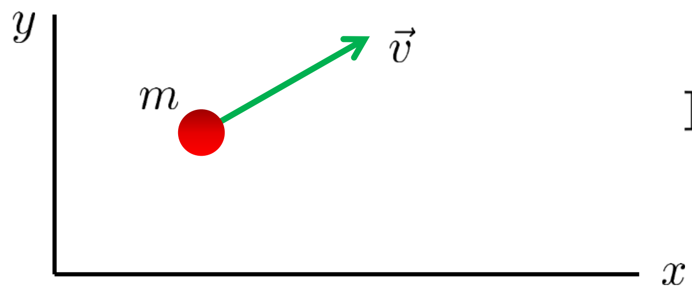
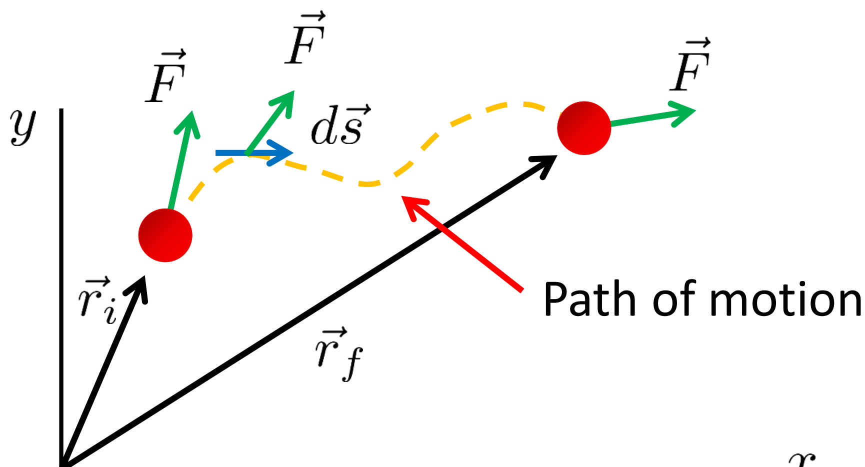
9/10-2: A Quick Recap of the Last Class



$$\textbf{Kinetic Energy}, \; K \equiv \frac{1}{2}mv^2$$

Work done by a Force:



General Expression:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s}$$

For motion along a line s:

$$W = \int_{s_i}^{s_f} F_s ds$$

For a Constant Force:

$$W = \vec{F} \cdot \Delta\vec{r}$$

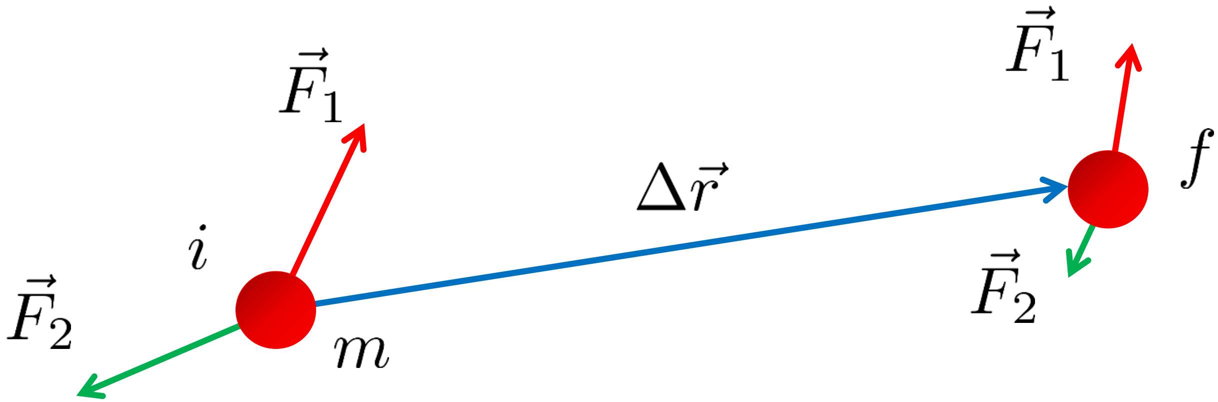
For a Constant Force parallel to the displacement:

$$W = F\Delta r$$

We're not going to be following the order of material in the text

Work – Kinetic Energy Theorem

OK, now let's get back to where we started with this Work stuff:
How do you change the kinetic energy of an object?

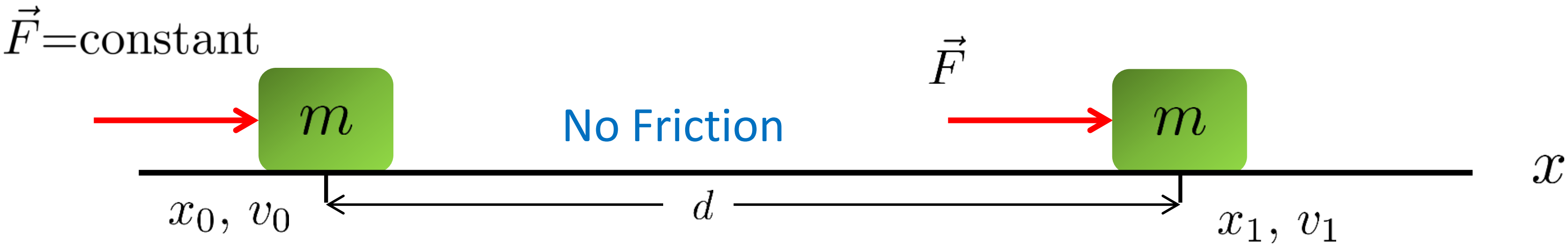


The work-kinetic energy theorem When one or more forces act on a particle as it is displaced from an initial position to a final position, the net work done on the particle by these forces causes the particle's kinetic energy to *change* by $\Delta K = W_{\text{net}}$. **(Your author also calls this "The Energy Principle")**

$$\Delta K = K_f - K_i = W_{net}$$

Does this make sense?
Let's take a look at a real simple problem for which we already know the answer.

A Simple Problem Done Two Ways



Know: x_0, v_0, x_1, F; what is v_1?

One approach, chapters 2 & 6:

Dynamics: $a = F/m$

Kinematics: $v_1^2 = v_0^2 + 2a\Delta x$

$0 \rightarrow 1$ $\quad (\Delta x = x_1 - x_0 = d)$

$$v_1 = \sqrt{v_0^2 + \frac{2Fd}{m}}$$

Another approach, Work-KE 0 to 1:

$$\Delta K = W_{net}$$

$$\frac{1}{2}m(v_1^2 - v_0^2) = \vec{F} \cdot d\hat{\imath} = Fd$$

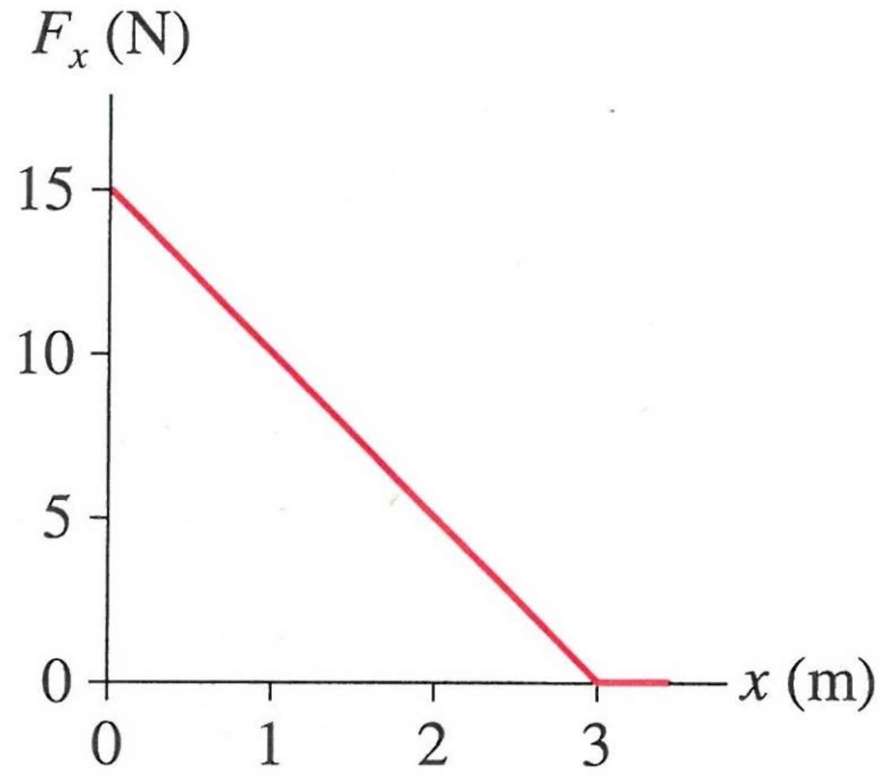
$$(v_1^2 - v_0^2) = \frac{2Fd}{m}$$

$$v_1 = \sqrt{v_0^2 + \frac{2Fd}{m}}$$

So, we have two ways to solve the same problem. Which one will we use? That will depend on the problem; in particular, what we know and what we want to find, and how much work (i.e. effort) we want to do.

Whiteboard Problem: 9/10-5

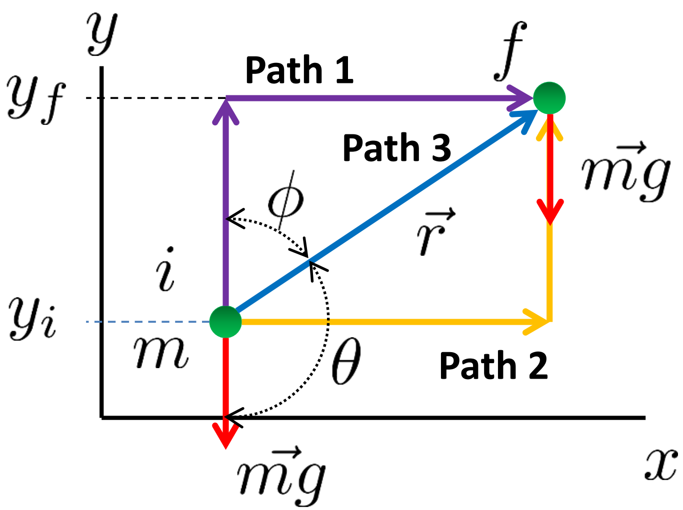
A 500 g particle moving along the x axis experiences the force shown below. The particle's velocity is 2.0 m/s at x = 0 m. **What is its speed at x = 3 m? (LC)**



Conservative Forces

Definition: A **Conservative* Force** is a force for which the work done by the force in going from some initial point *i* to some final point *f* **Does NOT depend on the path followed.**

By Example: Gravity:



Work done by Gravity going i to f:

$$\text{Path 1:}\ W_g = -mg(y_f - y_i)$$

$$\text{Path 2:}\ W_g = -mg(y_f - y_i)$$

$$\begin{aligned}\text{Path 3:}\ W_g &= \vec{m}g \cdot \vec{r} \\ &= mgr\cos\theta \\ &= -mgr\cos\phi \\ &= -mg(y_f - y_i)\end{aligned}$$

So the work done by gravity does not depend on the path, just the change in y. So gravity is a conservative force.

*As we'll see, the name comes from the fact that conservative forces conserve the total mechanical energy.

Potential Energy

Any Conservative Force can be associated with a Potential Energy:

For a system going from *an initial state i* to a *final state f* subject to a conservative force, **the change in potential energy is defined to be the negative of the work done by the conservative force.**

$$\Delta U = U_f - U_i \equiv -W_c(i \rightarrow f)$$

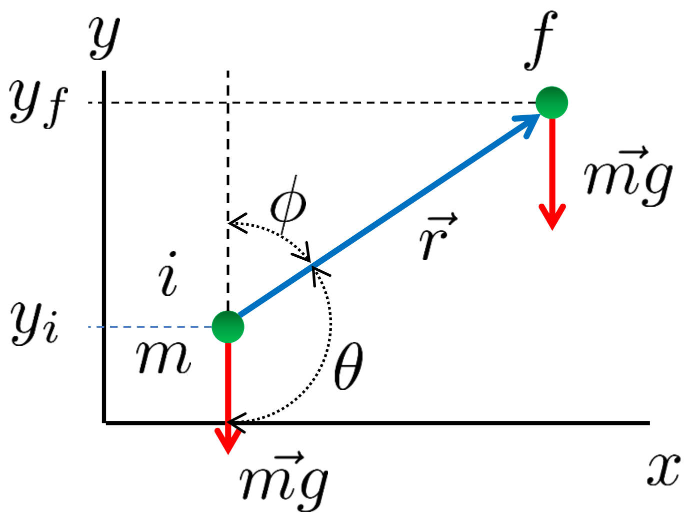
Subscript 'c' means work done by the conservative force

Since the work is path independent, the change in potential energy depends only on the initial and final points, not the path followed.

Note: we'll also get to Nonconservative Forces for which we can't associate a potential energy.

Gravitational Potential Energy

Gravity is a conservative force, so we can define a potential energy for gravity:



Work done by gravity in going from i to f:

$$
\begin{aligned}
W_g = \vec{mg} \cdot \vec{r} &= mgr\cos\theta \\
&= -mgr\cos\phi \\
&= -mg(y_f - y_i)
\end{aligned}
$$

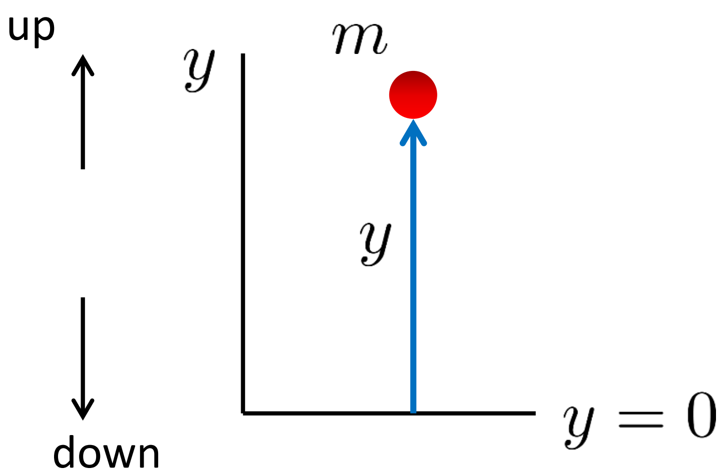
So, the Change in Gravitational Potential Energy is:

$$\Delta U_g = U_f - U_i = -W_g = mg(y_f - y_i)$$

And we say that the Gravitational Potential Energy depends just on the elevation:

$$U_g = mgy$$

More on Gravitational Potential Energy (PE)



*If you hold a mass above the floor, it has the **Potential** to move – if you release it. Or, it has the **Potential** to acquire Kinetic Energy – again, if you release it.*

*That's the origin of term **"Potential Energy"** which is associated with a position and has the "potential" to be converted to kinetic energy.*

$$\textbf{Gravitational Potential Energy}^*, U_g = mgy \text{ [Units = Joule]}$$

Important Note: you are free to set the $y = 0$ point and hence the $U_g = 0$ point anywhere you like. Only a change in PE (i.e. ΔU_g) has any physical meaning.

**This form for the gravitational PE is only valid near the surface of the Earth where g is fairly constant. We'll see a more general form in Chapter 13.*

Conservation of Mechanical Energy for Conservative Forces

Define: **Mechanical Energy = Kinetic Energy + Potential Energy**

$$\text{Or,} \qquad E_{mech} = K + U$$

It is the **Mechanical Energy** that is conserved in processes where there are no applied or dissipative forces like friction.

Now, the **Work – Kinetic Energy Theorem** for a system going from an initial state, i, to a final state, f, is:

$$\Delta K = W_{\text{net}}(i \to f) = W_c(i \to f) = -\Delta U$$

So, for problems where only conservative forces are present:

$$\boxed{\Delta E_{mech} = \Delta K + \Delta U = 0}$$

Where: $\Delta \overset{\text{(always)}}{=} \text{(final) - (initial)}$

In most problems, we'll have: $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$

(the only PE that have at this point) $\Delta U_g = mgy_f - mgy_i = mg(y_f - y_i)$

I start every conservation of energy problem with this equation (*or it's more general form, that we'll see soon*). It's different than what's in the text – ***why do I do it this way?*** **Every problem begins with exactly the same equation, even with multiple forms of PE. Also, including nonconservative forces, e.g. applied forces and friction, is quite easy and very straight forward – we'll do this in the next class.**

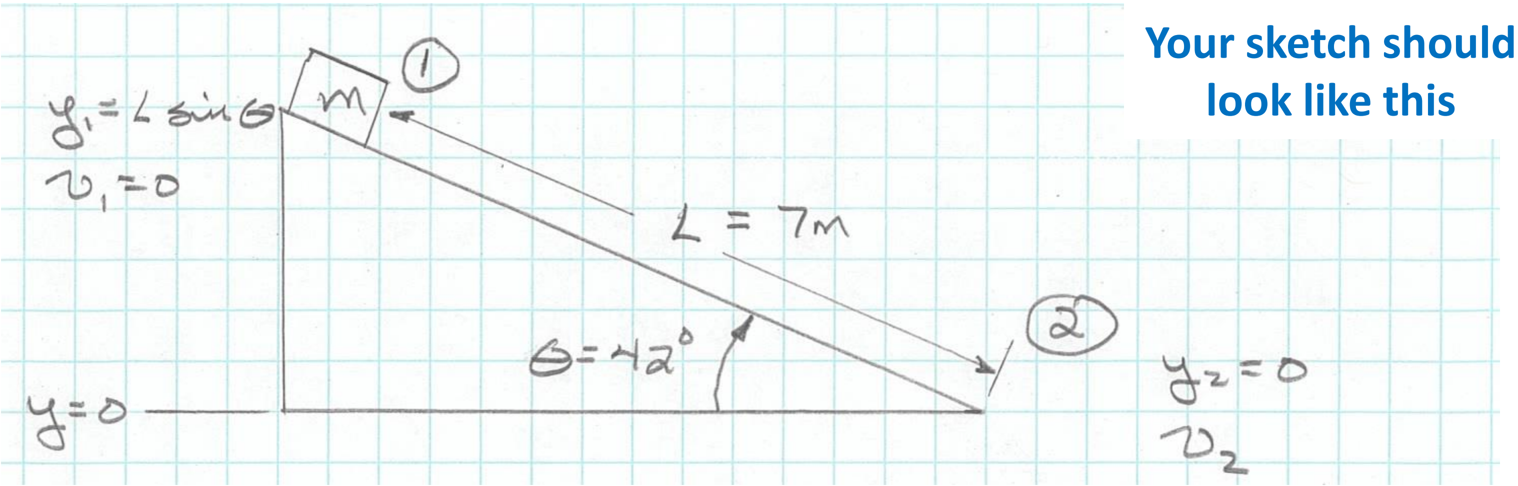
Whiteboard Problem: 9/10 - 6

A block of mass m is at the top of a smooth incline plane of angle 42° and length 7 m. The block is released from rest and slides to the bottom.

a) Sketch the problem.

b) What is the speed of the block at the bottom of the incline? (LC)

We know that you can do this problem with dynamics and 1D constant acceleration kinematics – we've done it dozens of times! Instead, use energy conservation to see how easy it actually is, and in a few minutes, we'll see problems where you can't easily use dynamics and kinematics.



Your sketch should look like this

Whiteboard Problem: 9/10 - 7

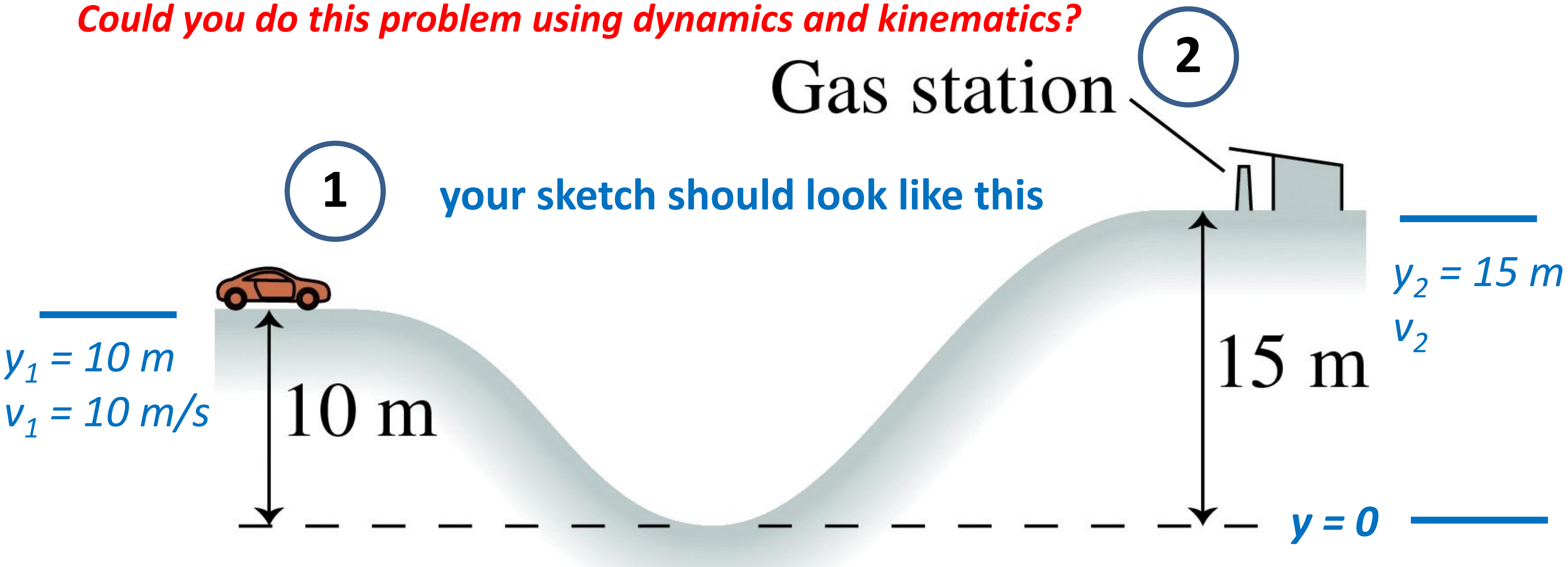
You're driving your car at a speed of 10 m/s on a level highway when it suddenly runs out of gas just as you get to a valley as shown. Alertly, you put the car in neutral hoping that it will roll to the gas station on the other side of the valley.

a) Sketch the Problem.

b) What is the car's speed when it rolls in to the gas station (LC)

(Ignore rolling friction and aerodynamic drag.)

Could you do this problem using dynamics and kinematics?



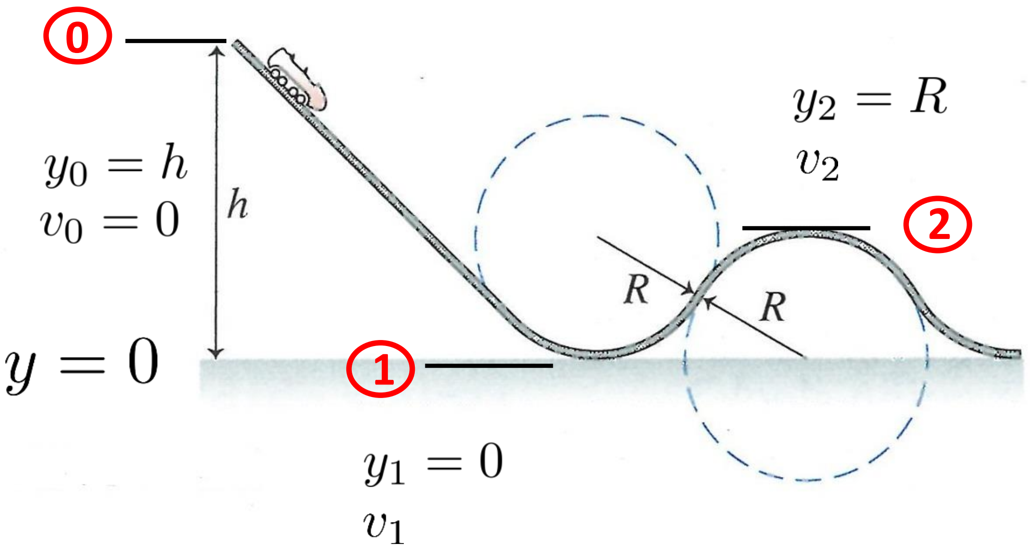
A Really Good (& fun) Whiteboard Problem: 9/10-8

A roller coaster car on a frictionless track shown below starts from rest at height h at point 0. The track's valley and hill consist of circular shaped segments of radius R.

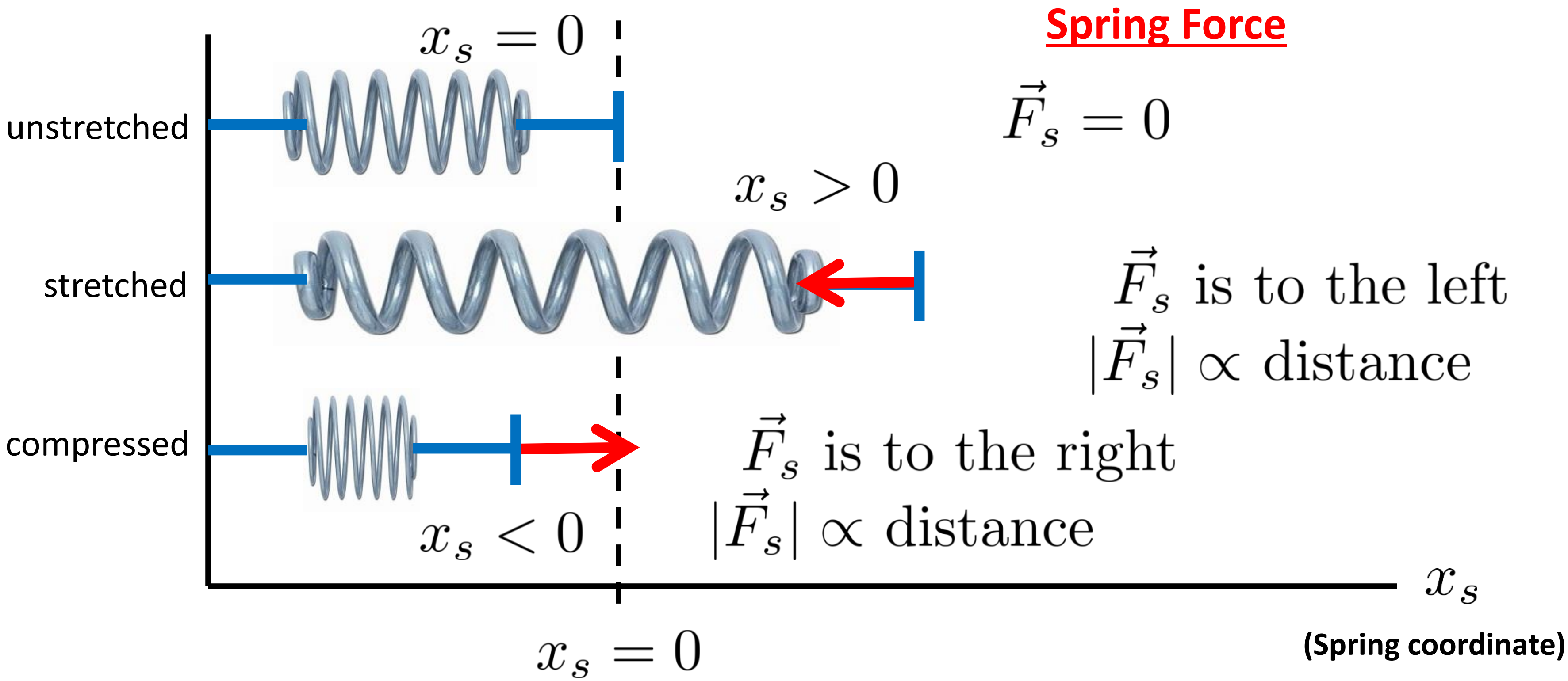
a) **What is the maximum height h_{max} (in terms of R) from which the car can start so as not to fly off the track going over the hill at point 2? (LC)**

b) **For the height found in part a, what is the apparent weight (in terms of the actual weight w) felt by the riders at the bottom of the valley at point 1? (LC)**

My sketch of the problem



Linear Springs – Hooke's* Law



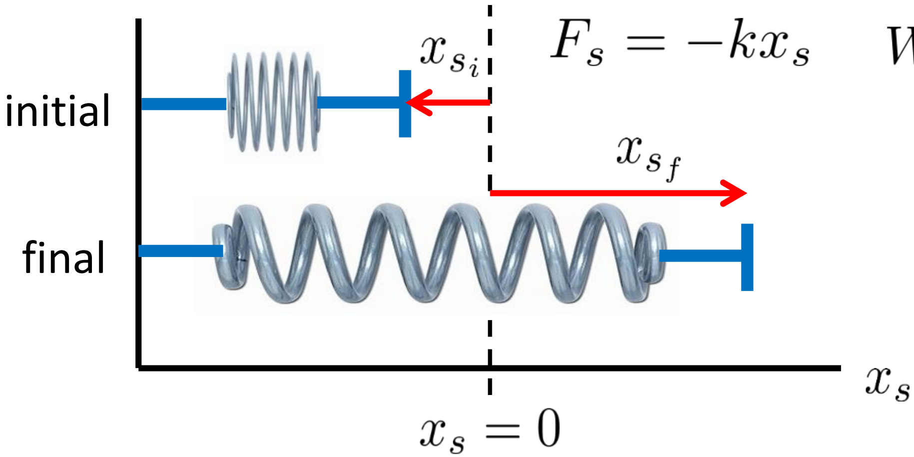
Component of the Spring Force: $F_s = -kx_s$

(for $x_s = 0$ at the equilibrium) $k =$ spring constant (≥ 0), [units, $\frac{N}{m}$]

Note: we call a force like this "***a linear restoring force***." (Why?)

**The same Hooke from the Newton film – his brother was the ship captain in Peter Pan. Hooke may have been presented as sort-of a jerk in the Newton's Dark Secrets video, but look at his [Wikipedia entry](); he made some impressive scientific contributions.*

Work & Potential Energy for a Linear Spring*



$$F_s = -kx_s$$

$$W_{s_{i\to f}} = \int_{x_{s_i}}^{x_{s_f}} F_s dx_s = \int_{x_{s_i}}^{x_{s_f}} -kx_s dx_s$$

$$= -k \int_{x_{s_i}}^{x_{s_f}} x_s dx_s = -k \left(\frac{x_s^2}{2} \right)_{x_{s_i}}^{x_{s_f}}$$

$$W_{s_{i\to f}} = -\frac{1}{2}k \left(x_{s_f}^2 - x_{s_i}^2 \right)$$

Is the Spring Force Conservative? **Yes**, the work done by F_s does not depend on the path taken between x_{si} and x_{sf}. (Try it)

So, we can define a change in the <u>Spring Potential Energy</u>:

$$\Delta U_s = -W_{s_{i\to f}} = \frac{1}{2}k \left(x_{s_f}^2 - x_{s_i}^2 \right)$$

<u>Spring Potential Energy</u>*

$$U_s = \frac{1}{2}kx_s^2$$

For $x_s = 0$ at the spring equilibrium

***<u>Note</u>,** We disagree with your author here. He writes his spring force and potential energy with the spring coordinate at some arbitrary point. <u>You can always choose the spring coordinate to be zero at the equilibrium</u>; it makes things a lot easier in using this in problems.

Whiteboard Problem 09/10 - 9

As F-18 fighters land on an aircraft carrier, the jet's tail hook snags a cable to slow it to a stop.
The cable is attached to a spring with spring constant 60,000 N/m. If the spring stretches 30 m to stop the fighter, and the fighter's mass is 15,000 kg, what was the plane's landing speed? **(LC)**

Like this:

My sketch: