9/10-1: Some Remarks about Part II of the Text

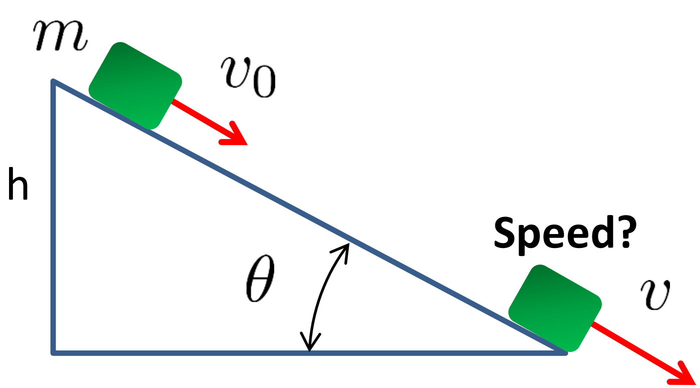
Up to this point in Physics 181, we have concentrated on:

Kinematics: How things move $\vec{v}(t) = \frac{d\vec{r}(t)}{dt}$ and $\vec{a}(t) = \frac{d\vec{v}(t)}{dt}$

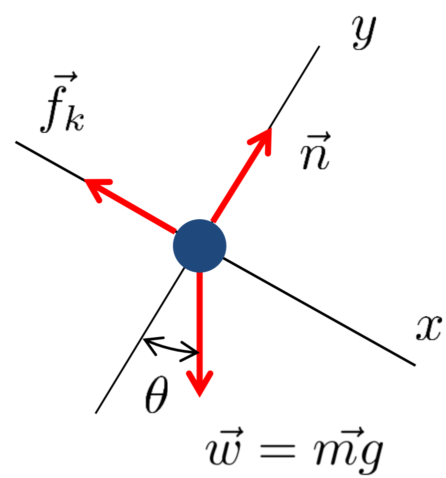
Dynamics: Why things move $\vec{F} = m\vec{a} = m\frac{d^2\vec{r}}{dt^2}$

(So, all of mechanics is nothing more than solving 1st and 2nd order differential equations!)

Here's something we can handle:



FBD:



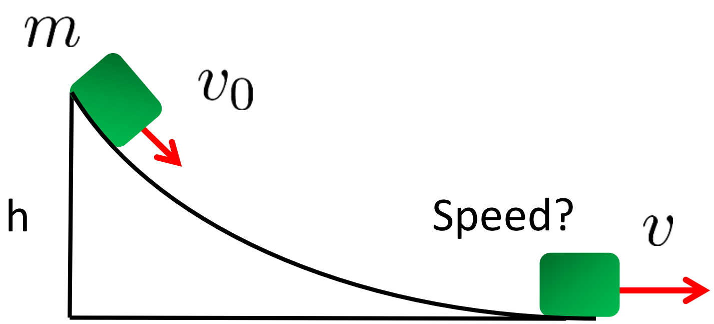
As we've done many times, for no friction: $a_x = g\sin\theta$

Or, if there is friction: $a_x = g(\sin\theta - \mu_k \cos\theta)$

Either way, for a_x = constant, we can solve for the speed

Some Remarks about Part II of the Text

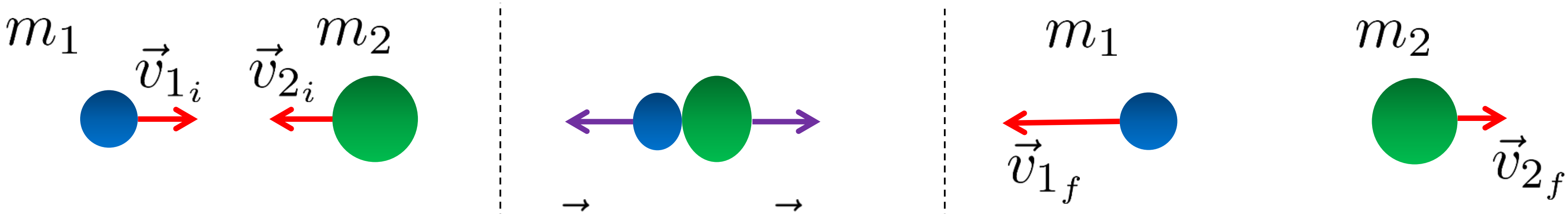
But, what if we had this problem:



Here, the FBD is not the same at every point on the slide.

This means that the **acceleration is not constant.**

Or, how about this, a collision between two objects:



Force on m_1 and $m_2 = 0$

$$\vec{F}_{2\text{on}1} - \vec{F}_{1\text{on}2}$$

(force is some complicated function of time - watch)

Force on m_1 and $m_2 = 0$

Find: $\vec{v}_{1_f}$ and $\vec{v}_{2_f}$

Both of these examples can be solved using Newton's Laws, but they're very very difficult! We need a better way. **Fortunately, there is one, and it's really easy!**

Some Remarks about Part II of the Text

Remember, we started Physics 181 with the observation:

"Everything moves"

And we've spent the entire semester learning how to describe that motion.
We're now ready to add to that observation:

"Everything moves, but in all processes, some quantities stay the same, i.e. are conserved."

A **Conservation Law** tells us that *something* stays the same, and we can use that to solve many types of problems very easily. The trick is to find out what that something is. Here, we'll just tell you what they are.

We'll concentrate on the two conservation laws:

Conservation of Energy (Chapters 9 & 10)

Conservation of Momentum (Chapter 11)

These will become essential tools that we can use to solve all kinds of problems that would otherwise be very difficult.

One Last Remark about Part II of the Text

Your author points out that these conservation laws are actually more fundamental than Newton's Laws*; this is true:

In the realm of the very small (i.e. atoms), Newton's 2^{nd} Law fails, but Energy and Momentum conservation are still valid:

Quantum Mechanics (end of PHY181)

Also, in the realm of the very fast (near the speed of light), both Newton's Laws and the rules of kinematics fail, but with more complete definitions of momentum and energy, the conservation laws are still valid:

Special Relativity (end of PHY182)

**Newton touched on the idea of momentum, but didn't do anything at all with energy.*

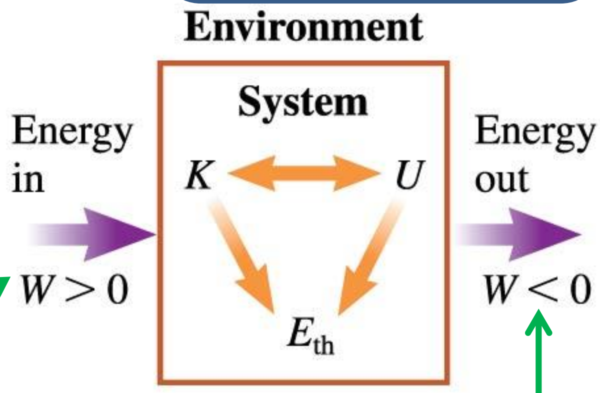
MODEL 9.1

Basic energy model (For Mechanical Systems)

Kinetic Energy, K
Potential Energy, U
Thermal Energy, E_{th}
$E_{sys} = K + U + E_{th}$

Energy is a property of the system.

- Energy is *transformed* within the system without loss. (i.e. $K + U + E_{th}$ = constant)
- Energy is *transferred* to and from the system by forces from the environment.
  - The forces do *work* on the system.
  - $W > 0$ for energy added.
  - $W < 0$ for energy removed.
- The energy of an *isolated system*—one that doesn't interact with its environment—does not change. We say it is *conserved*.
- The energy principle is $\Delta E_{sys} = W_{ext}$.
- Limitations: Model fails if there is energy transfer via thermal processes (heat).

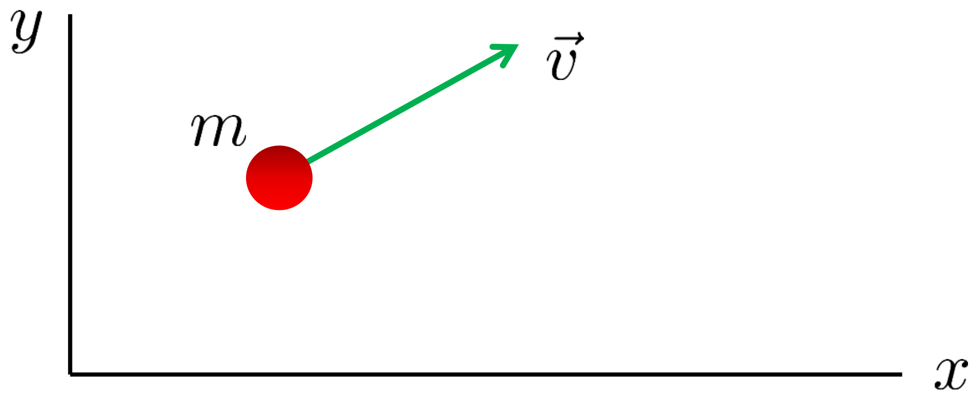


Exercise 1

We'll develop and refine this model as we go through Chapters 9 & 10, and we'll come back to it in PHY182 when we'll include thermal energy and heat.

Energy – the Basics- Kinetic Energy

Kinetic Energy = The Energy of Motion.



$$\textbf{Kinetic Energy},\ K \equiv \frac{1}{2} m v^2$$

$$\text{Units: } 1\ \frac{kg\ m^2}{s^2} \equiv 1\ Joule\ (J)$$

If Newton didn't use the idea of energy, where did the idea of kinetic energy come from? Watch this short video about **Emilie du Chateler** **(4 LC)**

Note: **Kinetic Energy (KE) is a Scalar**. Also, the v in the equation is the speed of the object. **Therefore, KE is always a positive quantity or zero.**

Another Note: As we'll see soon, you can obtain the Kinetic Energy from the velocity vector as :

$$K = \frac{1}{2} m\ \vec{v} \cdot \vec{v}$$

Whiteboard Problem: 9/10-1

a) At what speed does a 1000 kg compact car have the same kinetic energy as a 20,000 kg truck going 25 km/h? (LC)

b) A mother has four times the mass as her young son. While running, they have the same kinetic energy. What is the ratio of their speeds, v_{son}/v_{mother}? (LC)

The Concept of Work – 1D

How do you make an object speed up or slow down, **i.e. change it's kinetic energy**?
In the language of Chapter 6, you exert a Force on it.
In the language of Energy, you do work on it: positive work increases the kinetic energy, i.e. the speed; negative work decreases the kinetic energy, i.e. the speed.

Consider this: a constant force in 1D:

$$\text{What is } \Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 \text{ ?}$$

Kinematics and Dynamics:

$$v_1^2 = v_0^2 + 2a_x\Delta x$$

$$v_1^2 - v_0^2 = 2a_x(x_1 - x_0)$$

$$\underbrace{\tfrac{1}{2}m(v_1^2 - v_0^2)}_{\Delta K} = \underbrace{ma_x}_{F_x}(x_1 - x_0)$$

$$\Delta K = F_x(x_1 - x_0)$$

Your author does this for a 1D variable force:

(details on p. 222)

$$\Delta K = \int_{x_i}^{x_f} F_x dx = \text{Work}$$

i.e. The Change in KE = "Force x displacement" = **The Work done by the Force**

Note of caution: Work = Force x displacement is a very special case. It is valid only when the force is constant and parallel to the displacement.

The Concept of Work – 2D

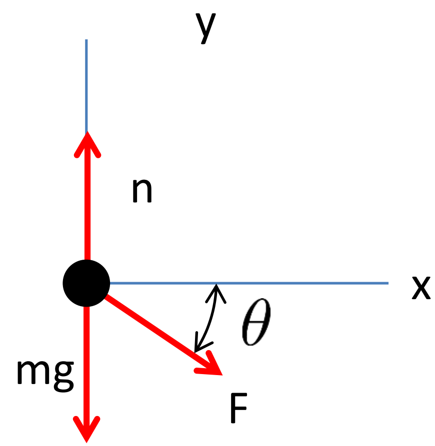
What about when we have a force in two dimensions?

Consider a block being pushed across a smooth floor with this constant force.

Now, how do we find the change in kinetic energy of the block?

We have to find the acceleration. **How about a FBD:**

$$\sum F_x = F\cos\theta = ma_x \Rightarrow a_x = \frac{F}{m}\cos\theta$$



Now, do the Kinematics:

$$v_1^2 = v_0^2 + 2a_x\Delta x$$

$$v_1^2 - v_0^2 = 2a_x(x_1 - x_0)$$

$$\frac{1}{2}m(v_1^2 - v_0^2) = ma_x(x_1 - x_0)$$

$$\frac{1}{2}m(v_1^2 - v_0^2) = m\left(\frac{F}{m}\cos\theta\right)(x_1 - x_0)$$

$$\underbrace{\frac{1}{2}m(v_1^2 - v_0^2)}_{\Delta K} = \underbrace{(F\cos\theta)}_{F_x}(x_1 - x_0)$$

$$\Delta K = F_x(x_1 - x_0)$$

Chair Demo

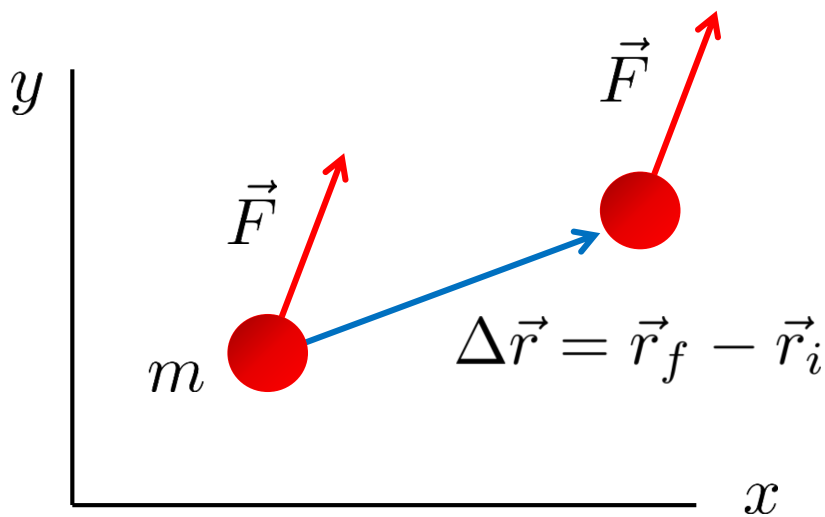
i.e. The Change in KE = "Component of Force parallel to displacement x displacement"

= **The Work done by the Force** (note: still for a constant force)

What is This Thing Called "Work"

(it seems pretty important)

Work done by a Constant Force:



Only the component of $\vec{F}$ parallel to $\Delta\vec{r}$ does any work.

$$\text{Work (done on } m \text{ by } \vec{F} \text{ moving } m \text{ by } \Delta\vec{r}) = \vec{F}\cdot\Delta\vec{r}$$

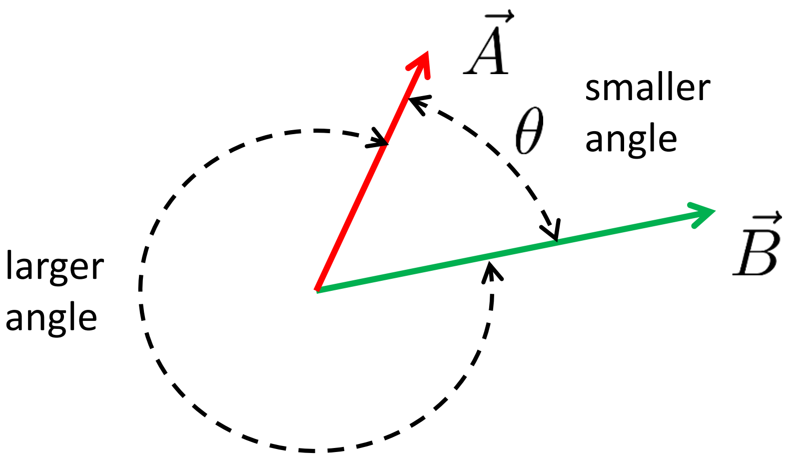
$$i.e. \quad \text{for } \vec{F} = \text{constant:} \qquad W = \vec{F}\cdot\Delta\vec{r} \qquad [\text{Units: } Nm = \text{Joule}]$$

But, what is this thing? **It's called a "Vector Dot Product"**

This is what finds the component of the force parallel to the displacement.

Vector Dot Product

(sometimes called a Scalar Product since it produces a scalar from 2 vectors)



$$\vec{A} \cdot \vec{B} = AB \cos\theta$$

Where: A and B are the magnitudes of vectors $\vec{A}$ and $\vec{B}$

θ is the smaller of the two angles between $\vec{A}$ and $\vec{B}$

So, the Dot Product produces a **scalar** that can be positive, negative, or zero.

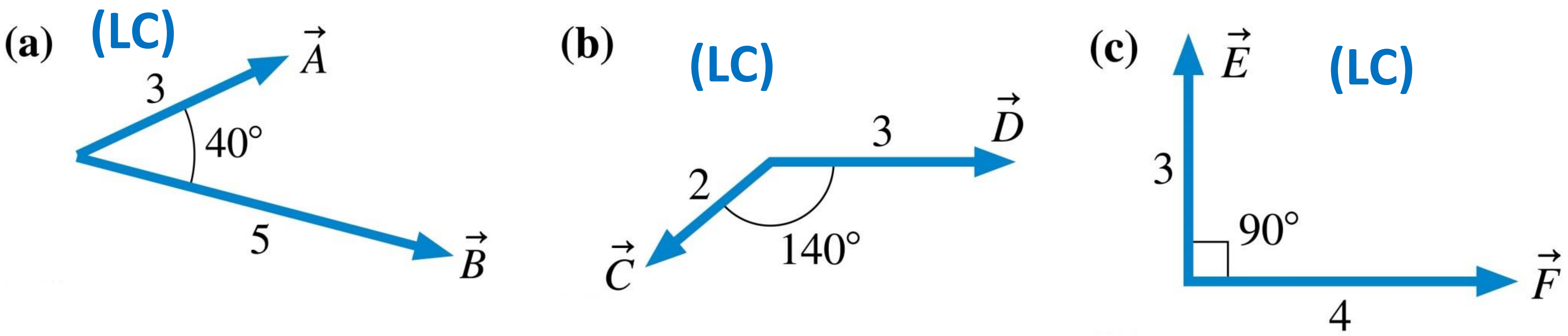
What does the dot product mean in words?

$\vec{A} \cdot \vec{B}$ is a measure of how much of $\vec{A}$ points in the direction of $\vec{B}$, and vice versa.

$\vec{A} \cdot \vec{B}$ goes from $-AB$ (vectors anti-parallel) to $+AB$ (vectors parallel) and $\vec{A} \cdot \vec{B} = 0$ for perpendicular vectors

Whiteboard Problem: 9/10-2

Evaluate the dot product of the following pairs of vectors:



Answer: 11.5

Answer: -4.60

Answer: 0.0

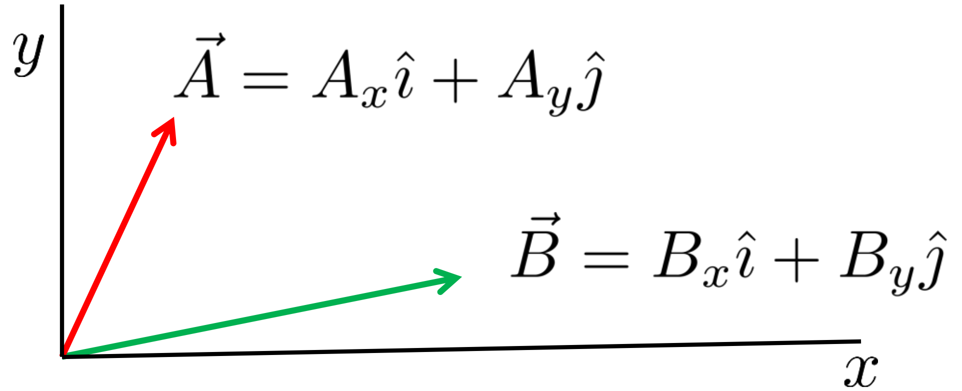
(d) Evaluate the dot product of:

$$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} \;\; \text{and} \;\; \vec{B} = 2\hat{\imath} - 6\hat{\jmath}$$

Difficult to find the angle . . . maybe we need another way to do a Dot Product?

Another Way to do Dot Products

Suppose: we know the components of the vectors relative to some reference frame:



It is easy to show that:

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$$

Note: The two formulae for the Dot Product are equally important and useful. If you know the angle between the vectors, you use one; if you know the components, you use the other – whatever is more convenient.
YOU HAVE TO KNOW AND BE ABLE TO USE BOTH FORMULAE!

Whiteboard Problem: 9/10-2 (continued)

(d) Evaluate the dot product of: (LC)

$$\vec{A} = 3\hat{\imath} + 4\hat{\jmath} \quad \text{and} \quad \vec{B} = 2\hat{\imath} - 6\hat{\jmath}$$

Answer:

$$\vec{A} \cdot \vec{B} = (3)(2) + (4)(-6) = -18$$

(e) Evaluate the dot product of: (LC)

$$\vec{D} = 3\hat{\imath} - 2\hat{\jmath} \quad \text{and} \quad \vec{F} = 6\hat{\imath} + 4\hat{\jmath}$$

$$\vec{D} \cdot \vec{F} = (3)(6) + (-2)(4) = 10$$

In fact, this gives us a useful way to find the angle between two vectors;
e.g. for the first two vectors in part (d) above:

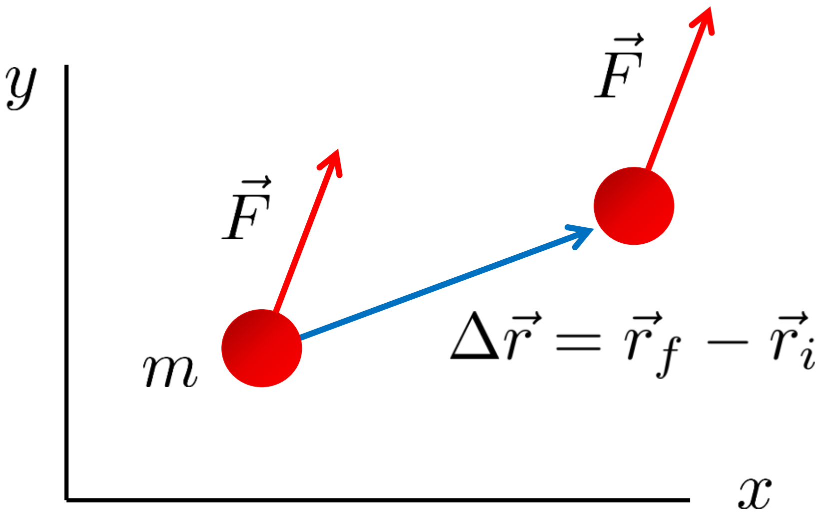
$$\vec{A} \cdot \vec{B} = -18 = AB\cos\theta$$

$$A = \sqrt{3^2 + 4^2} = 5.0 \qquad B = \sqrt{2^2 + 6^2} = 6.324$$

$$\text{So, } \theta = \cos^{-1}\left[\frac{-18}{(5)(6.324)}\right] = 124.7^\circ$$

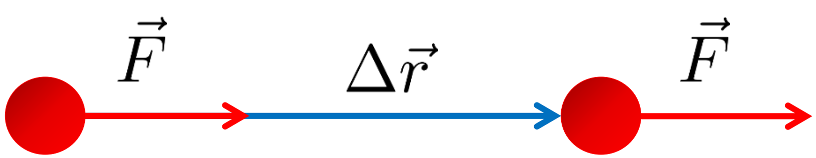
<u>Back to Work done by a Force</u>

For a <u>Constant Force</u>, we had:



$$W = \vec{F}\cdot\Delta\vec{r} \quad (\vec{F} = \text{constant})$$

Note: Now we see how **"Work = Force x Distance" is just a <u>special case</u>** for a constant force parallel to the displacement. Consider:



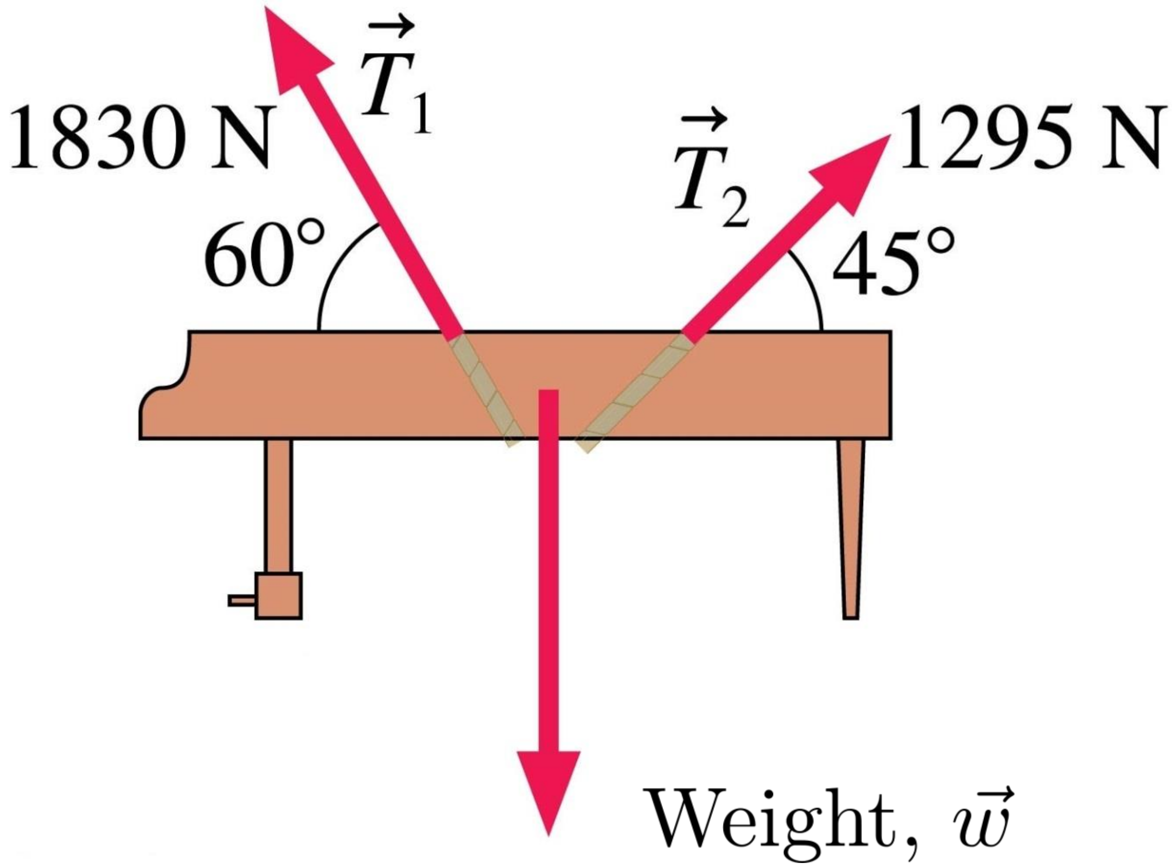
$$W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos(0^\circ) = F\Delta r$$

Whiteboard Problem: 9/10-3

The two ropes below are used to lower a 255 kg piano 5.0 m from a second story window to the ground at constant speed.

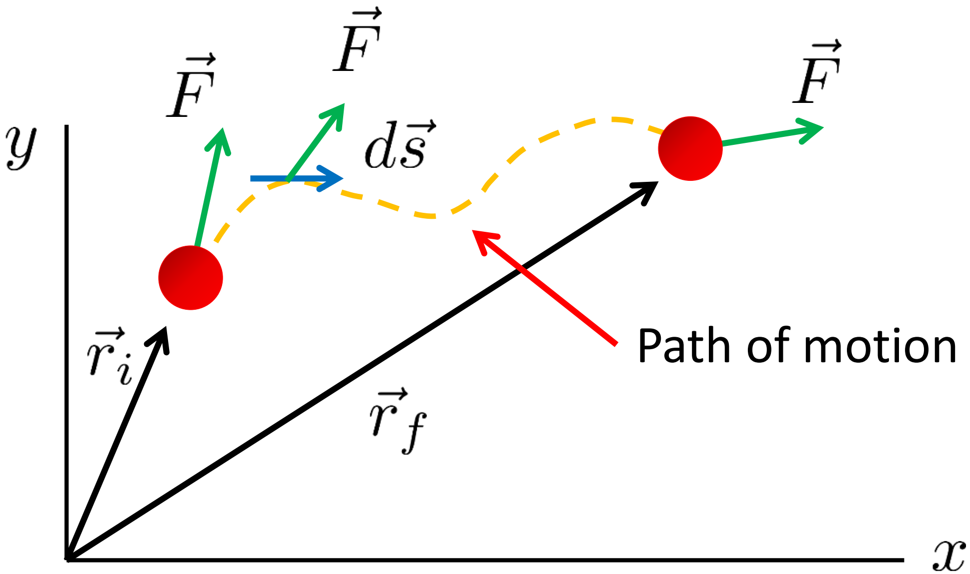
How much work is done by each of the three forces?

(LC, enter the work done by T_1)



Work done by a Variable Force

General Definition of Work done by a Force:



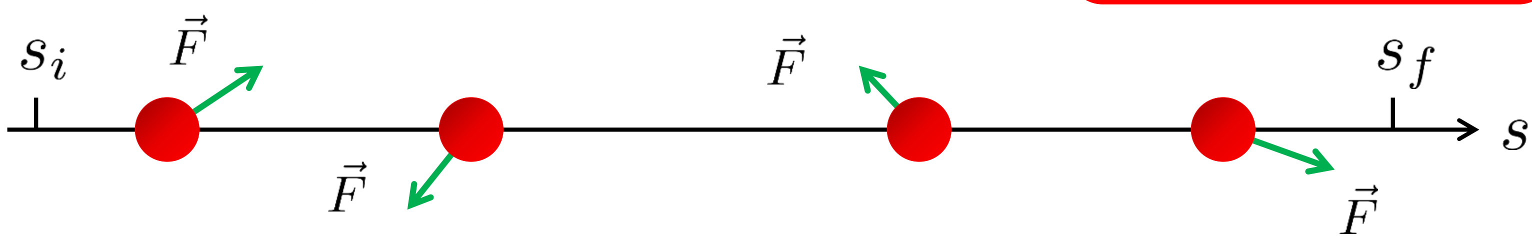
At every point along the path, an infinitesimal amount of work is done:

$$dW = \vec{F} \cdot d\vec{s}$$

So, just add these up for the total work done by F in going from the initial to the final point:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s}$$

For motion along a straight line:



Only that part of the Force parallel to the s-axis (i.e. the s-component) does any work.

$$W \text{ (by } \vec{F} \text{ going } s_i \text{ to } s_f) = \int_{s_i}^{s_f} F_s ds = \text{[area under the } F_s \text{ vs. } s \text{ curve]}$$

Whiteboard Problem 9/10-4

A person pushes horizontally on a heavy box and slides it across the level floor at constant velocity. The person pushes with a force of 75.0 N for the first 5.0 m, at which time he begins to tire. The force he exerts then starts to decrease linearly from 75.0 N to 0.0 N across the remaining 3.0 m.

How much work did the person do on the box? (LC)

(Hint: you might want to make a graph of force vs. distance)