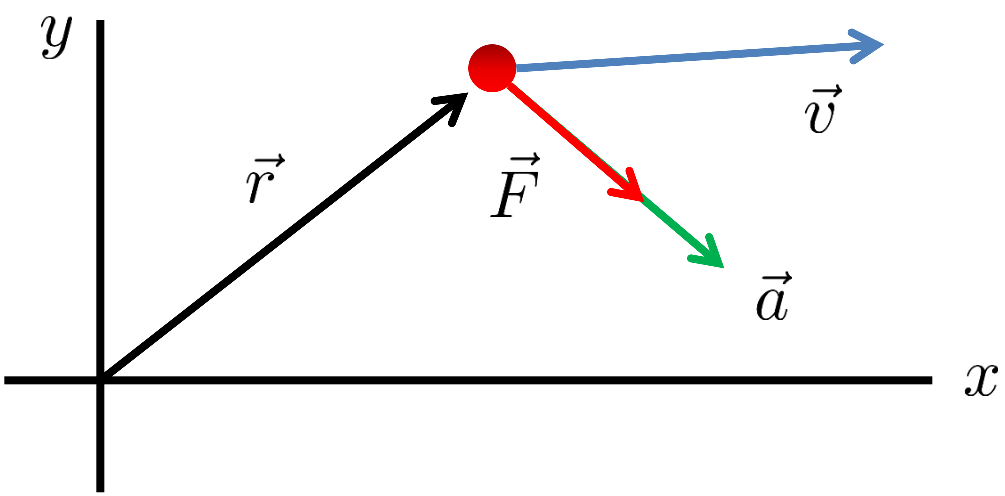
Chapter 8: 2D Dynamics

We have everything we need from chapters 4 and 6 to solve these problems:



Basic Equations:

$$\text{Kinematics: } \vec{v}(t) = \frac{d\vec{r}(t)}{dt} \qquad \text{and} \qquad \vec{a}(t) = \frac{d\vec{v}(t)}{dt}$$

$$\text{Dynamics: } \vec{F} = m\vec{a}$$

Of course, in practice, **we work with the component equations.**

2D Component Equations *(don't write these down; we already know them)*

For the vectors:

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} \qquad \vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} \qquad \vec{F} = F_x\hat{\imath} + F_y\hat{\jmath}$$

x-component equations:

$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

$$F_x = ma_x$$

y-component equations:

$$v_y = \frac{dy}{dt}$$

$$a_y = \frac{dv_y}{dt}$$

$$F_y = ma_y$$

And, for Constant Acceleration, the above equations can be integrated to give:

$$x_f = x_i + v_{x_i}\Delta t + \tfrac{1}{2}a_x(\Delta t)^2 \qquad y_f = y_i + v_{y_i}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$v_{x_f} = v_{x_i} + a_x\Delta t \qquad v_{y_f} = v_{y_i} + a_y\Delta t$$

These are the equations that we'll use.

$$v_{x_f}^2 = v_{x_i}^2 + 2a_x\Delta x \qquad v_{y_f}^2 = v_{y_i}^2 + 2a_y\Delta x$$

$$\Delta x = x_f - x_i \qquad \Delta t = t_f - t_i \qquad \Delta y = y_f - y_i$$

$$F_x = ma_x \qquad F_y = ma_y$$

Whiteboard Problem 8-1

A rocket powered hockey puck has a thrust of 2.0 N and a total mass of 1.0 kg. It is released from rest on a frictionless table, 4.0 m from the edge of a 2.0 m drop.

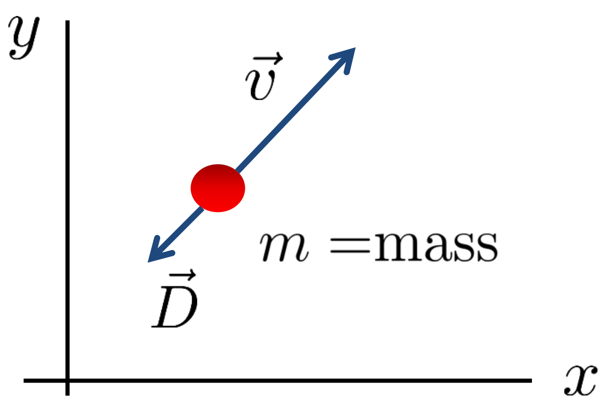
Ignoring aerodynamic drag, how far does the puck land from the base of the table? (LC)

What do we have to assume to solve this problem?

The puck does not rotate so that the thrust from the rocket remains in the same direction, i.e. horizontal.

Projectile Motion with Aerodynamic Drag

Recall from our **Aerodynamic Drag Model** from Chapter 6:



$$\vec{D} = \left(\tfrac{1}{2}\rho C A v^2,\ \text{direction } -\vec{v}\right)$$

$A =$ cross sectional area

$\rho =$ air density

$C =$ drag coefficient

Working with vector components for the velocity and the acceleration, it's fairly (?) easy to show that the x and y components of the acceleration that include gravity and aerodynamic drag can be expressed as:

$$a_x = -\frac{\rho C A}{2m} v_x \sqrt{v_x^2 + v_y^2}$$

$$a_y = -g - \frac{\rho C A}{2m} v_y \sqrt{v_x^2 + v_y^2}$$

Notice that these are coupled equations.

What does that mean?

a_x depends on v_x <u>and</u> v_y ; and a_y depends on v_y <u>and</u> v_x

Examples of Trajectories with Air Drag

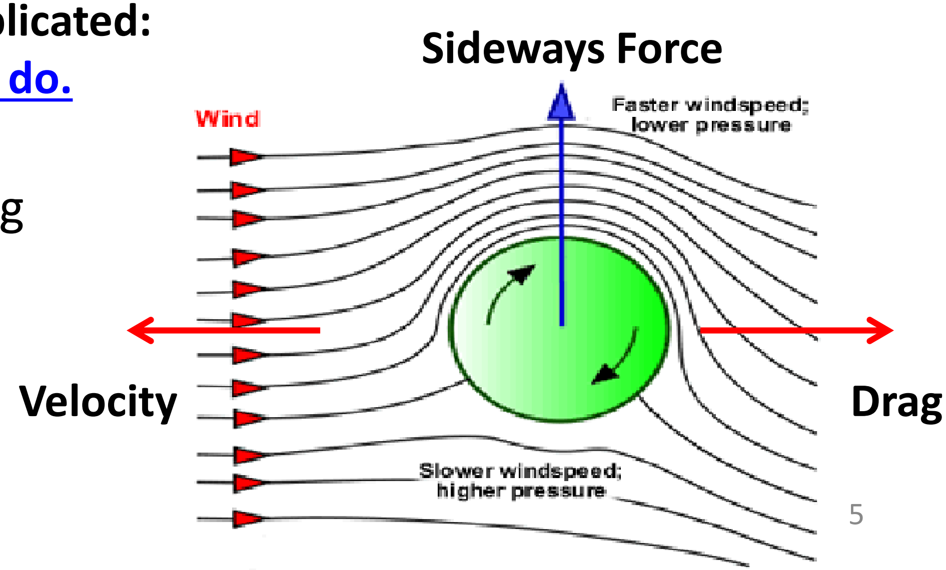
The equations above are what are solved numerically in the PhET simulation that you used when aerodynamic forces are turned on.

Some samples of actual golf ball trajectories including air drag:

How do they do that? It's even more complicated:
watch what PGA golfer Bubba Watson can do.

It's all in the spin of the ball which along with the dimples can create additional forces.

Or a baseball curving,
Or, even easier to see, a soccer ball.

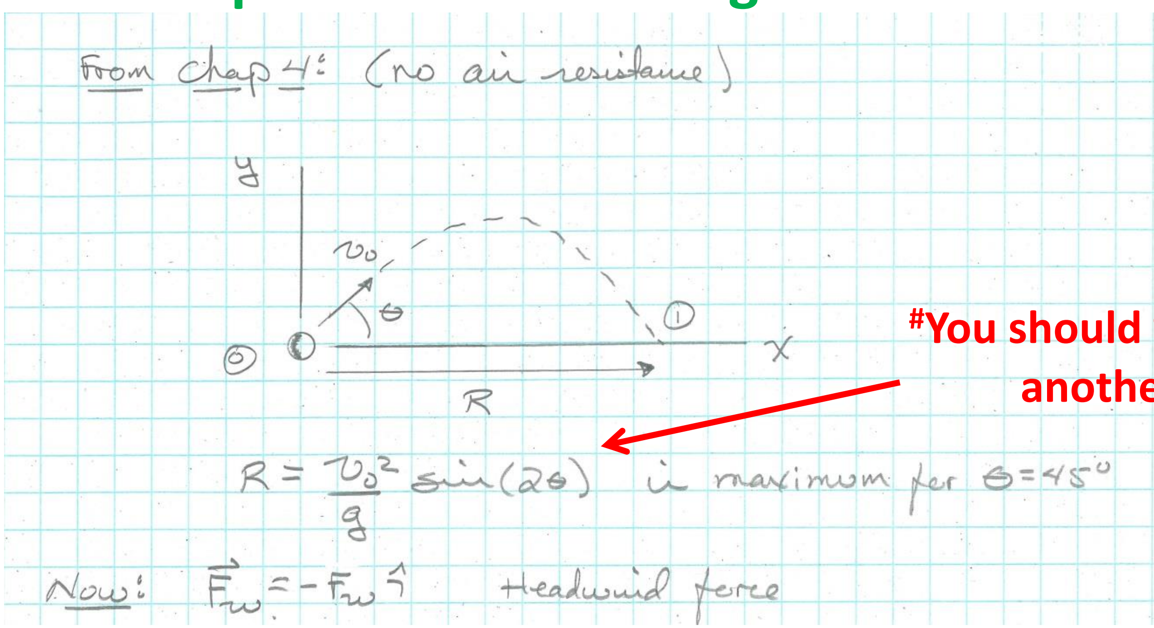


Whiteboard Problem: 8-2: *Kicking into the wind*

In the absence of air drag, a projectile that lands at the elevation from which it was launched achieves maximum range when launched at a 45° angle*. Suppose a projectile of mass m is launched with a speed of v_o and angle θ into a headwind that exerts a constant horizontal drag force: $\vec{F}_w = -F_w\hat{\imath}$

a) **Here's an example of this: a field goal kicked into the wind.**
b) **Find an expression for the "range" of the projectile#. (LC)**
c) **Try to find an expression for the angle for maximum range.**

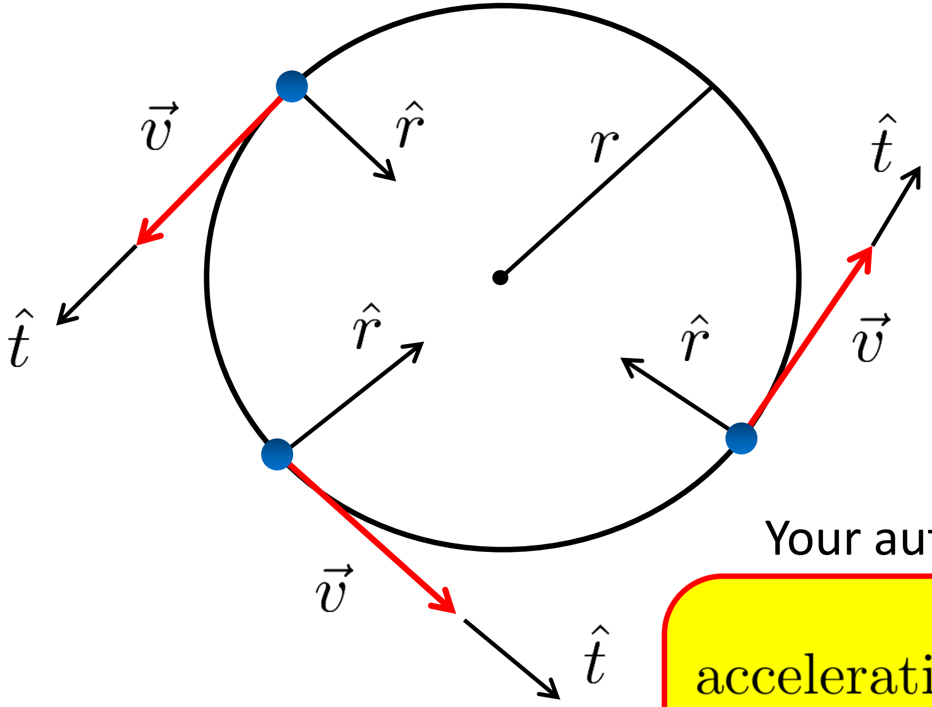
*Remember this from Chap 4:



#You should find this and another term.

Dynamics of Uniform Circular Motion

From Chap 4: for an object in uniform circular motion (UCM), the speed is constant, but the velocity continuously changes direction. **Thus, there must be an acceleration and a force that produces that acceleration.**



$\hat{r} =$ The instantaneous radial direction, Always toward the center of the circle

$\hat{t} =$ The instantaneous tangential direction, Always tangent to the circle.

For UCM:

$\vec{v}$ is in the tangential direction

$\vec{a}$ is in the radial direction

Your author shows in the text:

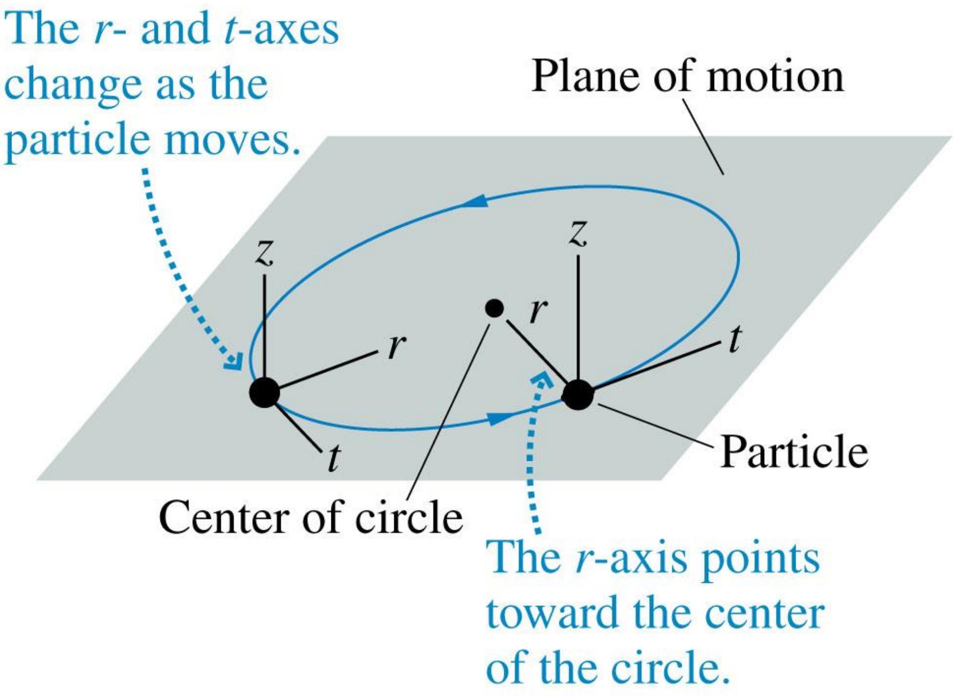
$$\text{acceleration, } \vec{a} = \left(\frac{v^2}{r}, \text{towards the center (or radially inward)} \right)$$

"Centripetal"

Valid for the radial component of acceleration even for nonuniform circular motion

Dynamics of UCM

To analyze UCM problems, we want to make full use of a convenient **rotating coordinate system** that your author calls the ***rtz-coordinate system***.



$\hat{r}$ = radial axis

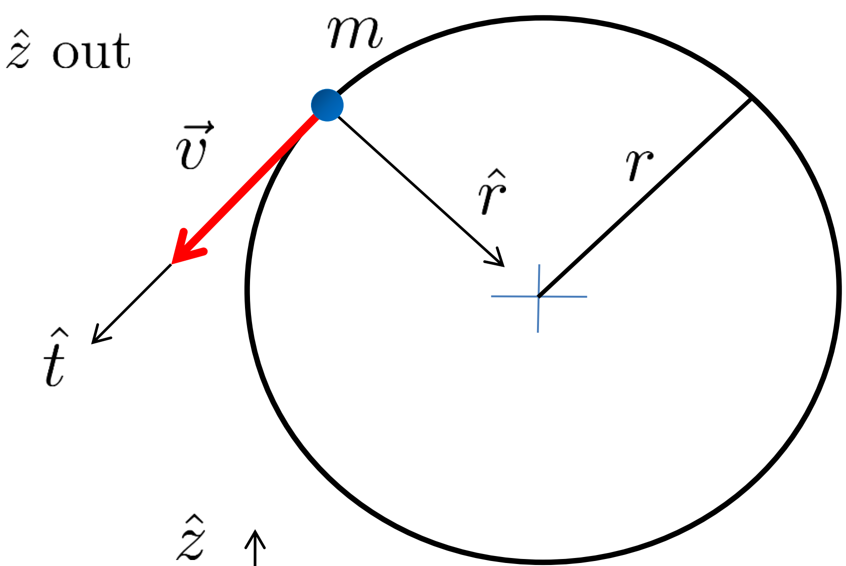
$\hat{t}$ = tangential axis

$\hat{z} \perp \hat{r}$ and $\hat{t}$

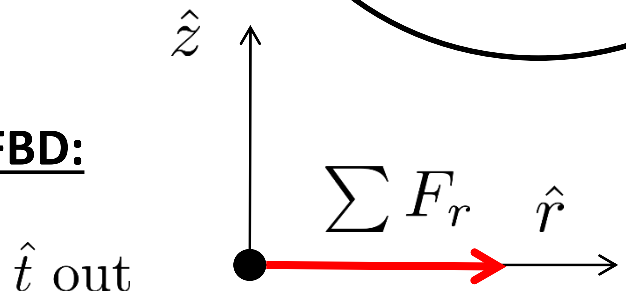
Note: for UCM problems, I put hats on the coordinate labels to distinguish them from other r, t, & z's that might be in the problem.

Also: for UCM problems, we aren't free to choose any coordinates; **the radial, or $\hat{r}$ direction, MUST always be toward the center of rotation.**

Dynamics of UCM



FBD:



For UCM, must have:

$$a_r = \frac{v^2}{r}$$

$$a_t = 0$$

$$a_z = 0$$

$$\sum F_z = ma_z = 0$$

$$\sum F_t = ma_t = 0$$

$$\sum F_r = ma_r = \frac{mv^2}{r}$$

Therefore, for UCM, there must be a **centrally directed force** (*the "centripetal force"*):

$$\vec{F} = \left(\frac{mv^2}{r}, \text{toward the center of rotation} \right)$$

But . . . This is not a new force! In problems, the centripetal force will be caused by familiar forces like friction, tension, normal force, etc. **e.g. swinging keys on a chain**.

Whiteboard Problem 8-3

A car of mass ***m*** drives around a flat circular track of radius ***r*** with a constant speed ***v***.

a) **Draw the Free Body Diagram of the car. What force is the centripetal force? (LC)**

b) **If** $\mu_s = \text{coefficient of static friction}$ **for rubber tires on concrete, find an expression for the maximum speed that the car can have and not slide off the track. (LC)**

Here's a guy on a motorcycle who didn't do this calculation.

What is providing the radial (or centripetal) force for this circular motion?
(What does the pilot feel during this turn?)

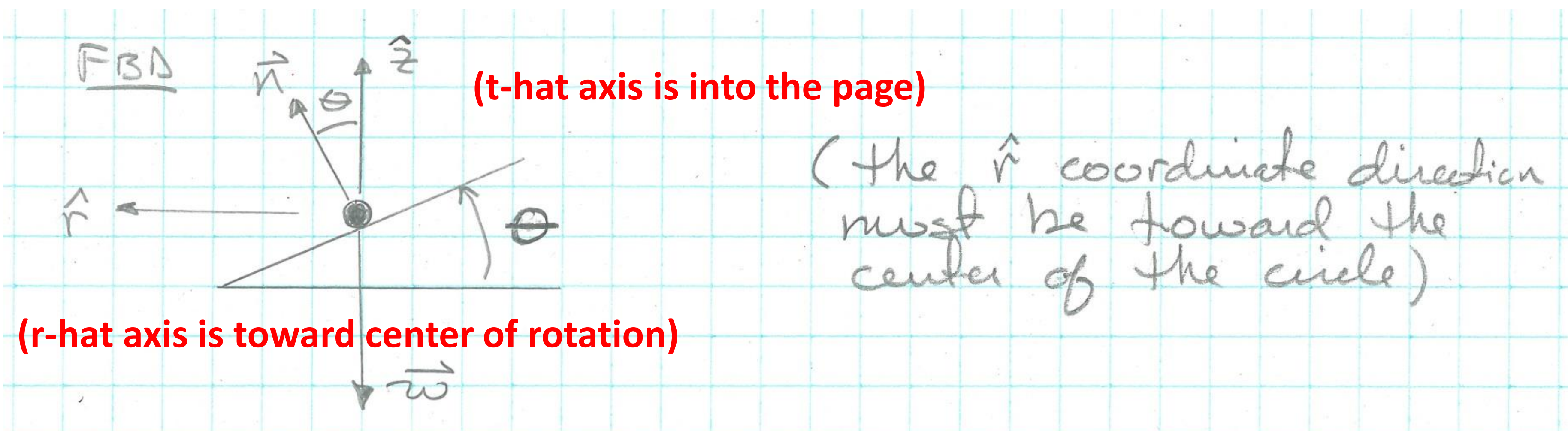
Whiteboard Problem 8-4

A highway curve of radius ***r*** is designed for traffic moving at a speed of ***v***. *Cars should be able to go around the curve at this speed even if the highway is covered with ice, and there is no friction!* This can be accomplished by banking the curve at some angle θ.

a) Draw a Free Body Diagram of a car going around this frictionless curve. What is the centripetal force? (LC)

b) Find an expression for the correct banking angle of the road? (LC)

Your FBD should look like this:



The r-component of the normal force is the centripetal force

Whiteboard Problem: 8-5 A Real Highway Curve

A car goes around a highway curve of radius r and bank angle θ. The coefficient of static friction between the tires of the car and the road is μ_s.

a) **Draw a Free Body Diagram of the car going around the curve. What is the centripetal force? (LC)**

b) **Find an expression for the maximum speed that the car can have to go around this curve without sliding. (LC)**

(your answer should be in terms of r, g, μ_s, and θ)

Your FBD should look like this:

(t-hat axis is into the page)

The r-components of the static friction and the normal are the centripetal force.