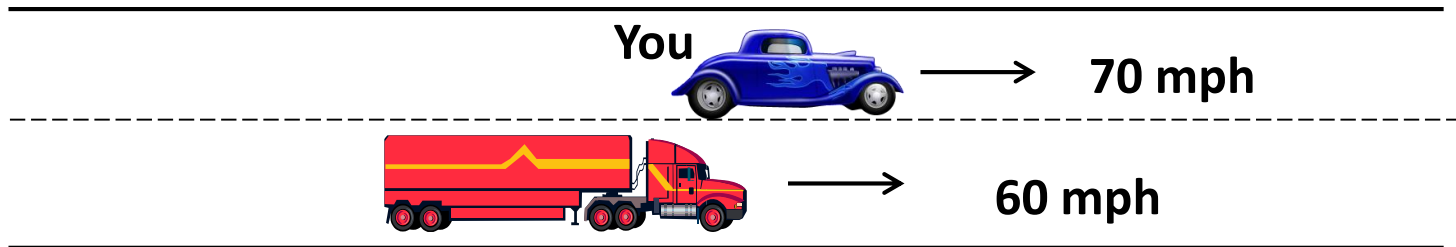


4-2: Relative Motion

(a little more than what's in your text, so pay attention)

Relative motion is something we use everyday, but we don't really think about it.

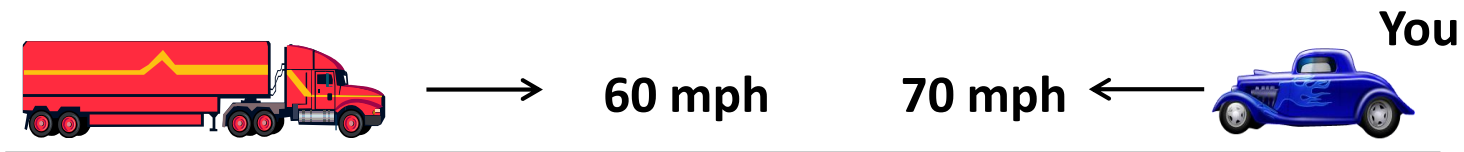
For example, passing a truck on the highway:



What is your speed relative to the truck? (*Or more precisely, what does someone on the truck measure for your speed?*)

10 mph

Or:



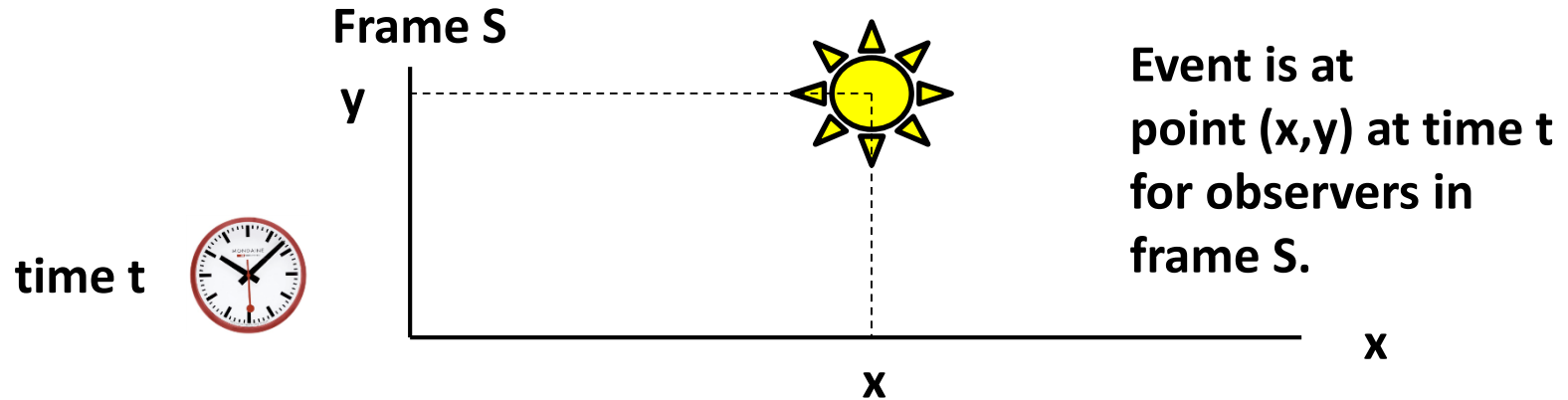
What is the truck's speed relative to you? (*Or more precisely, what do you measure for the truck's speed? What about velocities?*)

130 mph

These questions are pretty easy in 1D, but in 2D, it's better to follow a procedure.

Events and Frames of Reference

Event: something that happens at a particular place in space at a particular time.

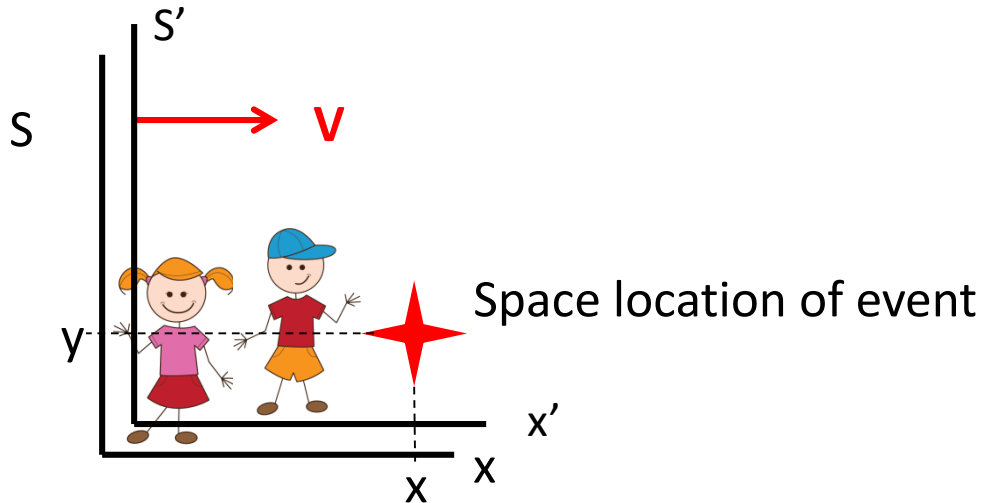


Reference Frame: coordinate systems and clocks to measure position and time of an event.

Relative Motion is concerned with how coordinates of events and velocities of objects are related in two frames of reference that are moving relative to each other.

Visualization

A boy is moving with constant velocity V relative to a girl. Let the girl be frame S and the boy be frame S' . Their frames correspond at $t=0$. They each observe the location of some event.



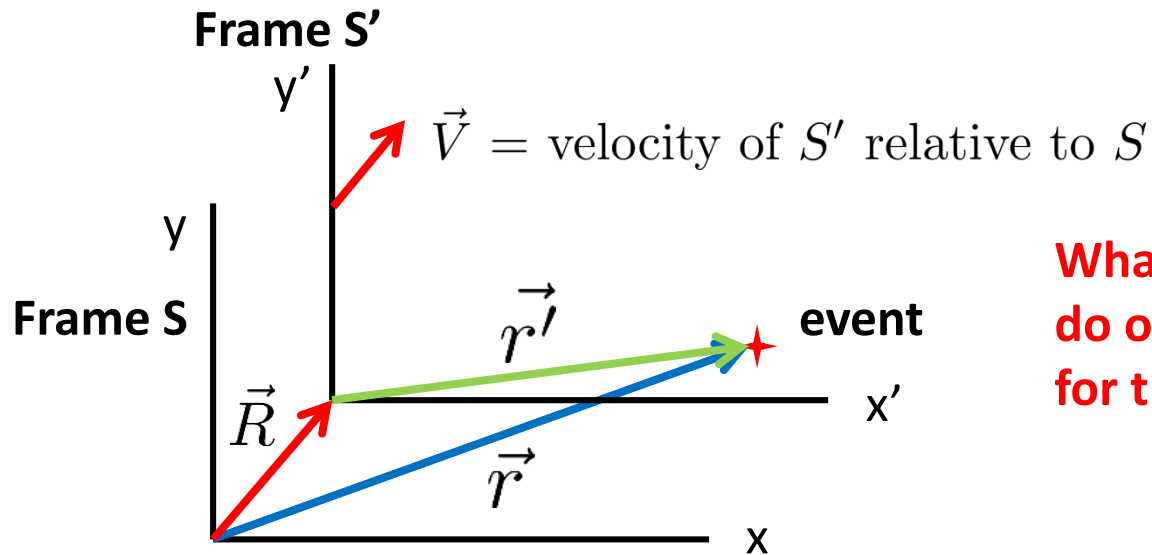
At $t=0$, an event has the same coordinates in frames S and S' , but what about an event at later times when S' is moving with constant velocity?

The x' coordinate of the event as observed by the boy will change.

(Note the "prime" in S' has nothing to do with differentiation)

(visualization won't work in a pdf)

Relative Motion



**What coordinates and time (?)
do observers in S and S' measure
for the event?**

\vec{r} = position vector of the event relative to S

\vec{r}' = position vector of the event relative to S'

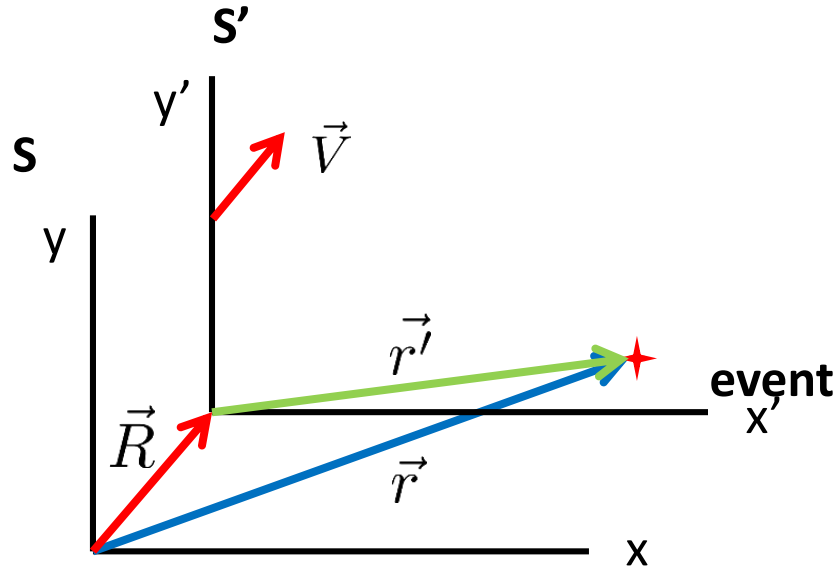
Let: Frames S and S' coincide at $t=0$; and x and x' axes be parallel

\vec{V} = constant velocity of S' relative to S

So, \vec{R} = position vector of S' relative to S and $\vec{R} = \vec{V}t$

Finally, from vector addition: $\vec{r} = \vec{r}' + \vec{R} = \vec{r}' + \vec{V}t$

The Galilean Position Transformations



**interchange primes and unprimes, and $\vec{V} \rightarrow -\vec{V}$*

From the previous page:

$\vec{r} = \vec{r}' + \vec{V}t$

Or in component form:

$x = x' + V_x t$

$y = y' + V_y t$

Obviously(?), S and S' measure the same time, so $t = t'$

Inverse*

$\vec{r}' = \vec{r} - \vec{V}t$

$x' = x - V_x t$

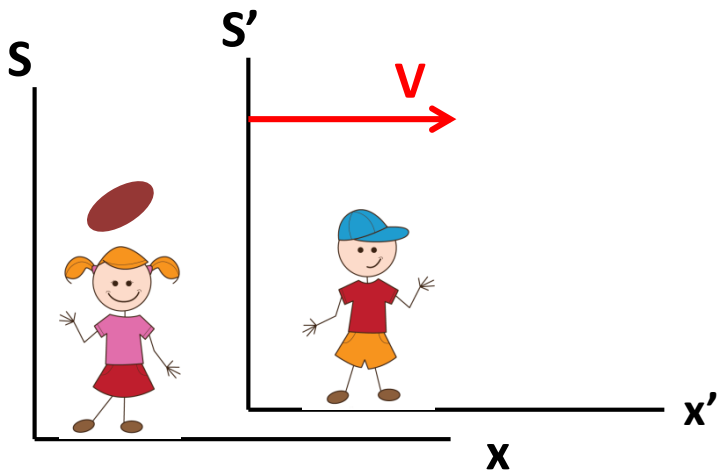
$y' = y - V_y t$

“Who says that the time is the same in the two frames?” Wait until PHY182.



Relative Velocity Visualization

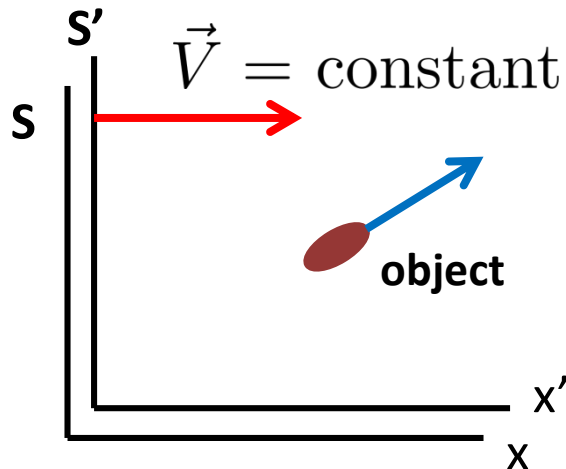
Consider a moving football pass as seen from two reference frames where one is in motion relative to the other frame.



What do observers in frames S and S' measure for the velocity of the football? The boy and girl will measure different velocity components for the ball, and hence will observe a different speed and direction.

(visualization won't work in a pdf)

Galilean Velocity Transformations



Let:

$$\vec{v} = \text{velocity of object relative to } S = \frac{d\vec{r}}{dt}$$

$$\vec{v}' = \text{velocity of object relative to } S' = \frac{d\vec{r}'}{dt}$$

The Galilean Position Transformation:

$$\vec{r} = \vec{r}' + \vec{V}t$$

Differentiate with respect to time:

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}'}{dt} + \frac{d}{dt}(\vec{V}t)$$

So:

$$\vec{v} = \vec{v}' + \vec{V}$$

Inverse

$$\vec{v}' = \vec{v} - \vec{V}$$

Or, in component form:

$$v_x = v'_x + V_x$$

$$v'_x = v_x - V_x$$

$$v_y = v'_y + V_y$$

$$v'_y = v_y - V_y$$

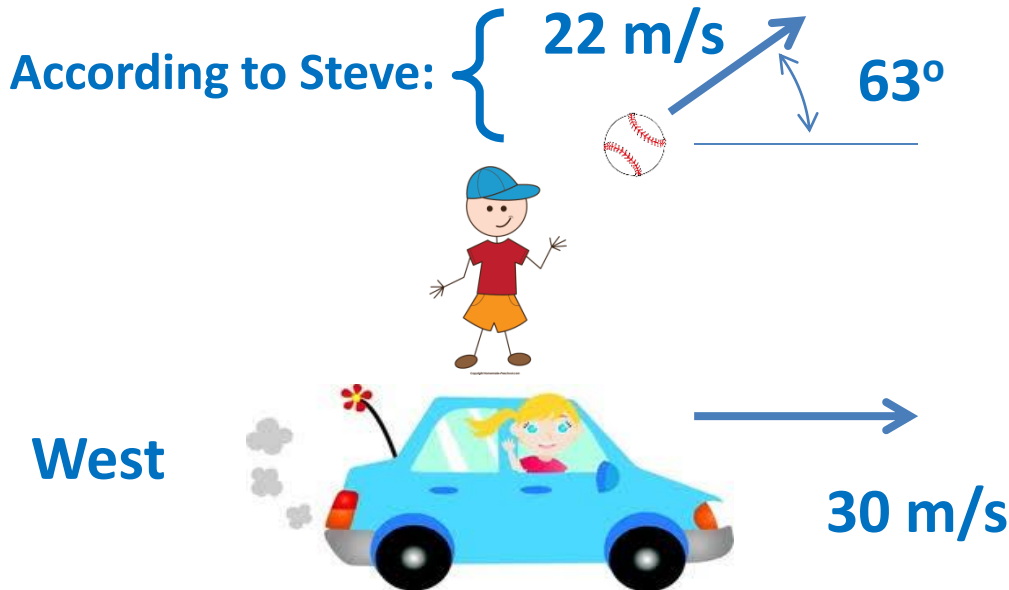
Galilean Velocity Transformations.

If we know the components of the velocity of an object in one frame, we can calculate them in another frame. See the equation sheet.

Whiteboard Problem 4-5

Steve throws a baseball, and according to him, it is moving upward and toward the East at a 63° angle above the ground with a speed of 22 m/s. Nancy is driving East past Steve at 30 m/s at the instant he releases the ball. At this instant,

- according to Nancy, what is the speed of the ball? (LC)
- according to Nancy, in what direction is the ball travelling? (LC)



Note: The most important step in doing any relative motion problem is to define the frames of reference and draw them. Then define what frames the velocities are relative to.

Whiteboard Problem 4-6: Driving in the Rain

What do you observe when you drive in the rain or snow? When the car is not moving, the rain may be coming straight down, but when you're driving, it appears to be coming at you.



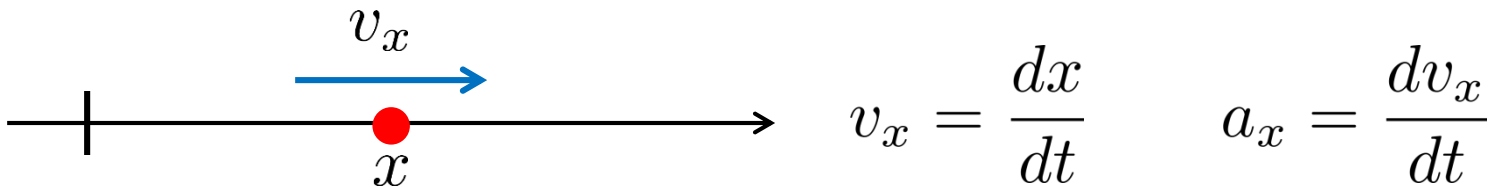
Suppose that when you're standing still, you observe that raindrops fall straight down with a speed of 10 mph. When you are driving in your car at a speed of 60 mph, what do you observe for the speed of the raindrops, and, according to you, what direction (LC) are they coming from?

Uniform Circular Motion (UCM)

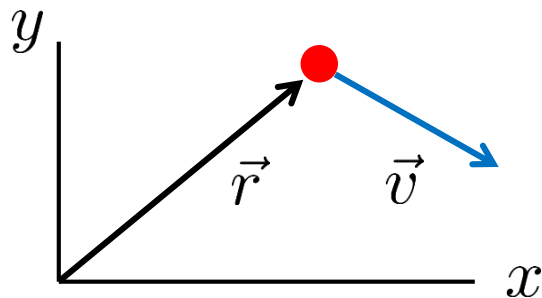
(remember, we won't be doing section 4.6 now)

A Quick Review:

So far, we've looked at **1D motion along the x-axis (Chapter 2):**



Then, we extended this to **2D (Chapter 4):**

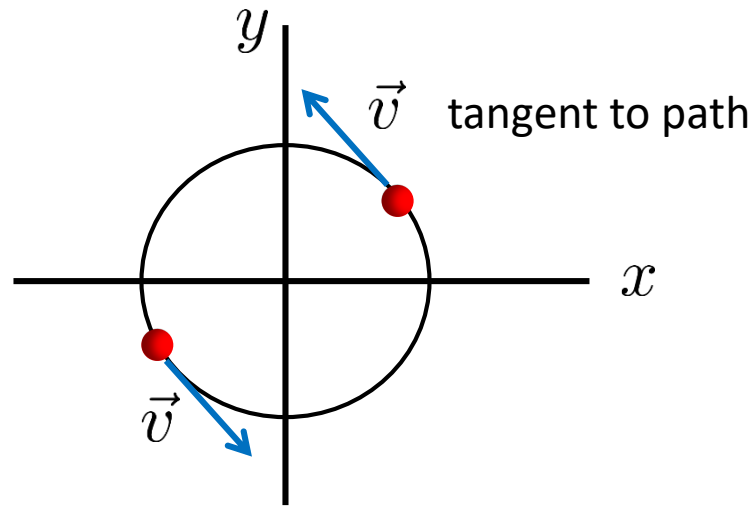


Or, in component form:

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \vec{a} = \frac{d\vec{v}}{dt}$$
$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt}$$
$$a_x = \frac{dv_x}{dt} \quad a_y = \frac{dv_y}{dt}$$

Uniform Circular Motion

Uniform Circular Motion is a special case of 2D motion where an object moves in a circular path at constant speed.



Note:

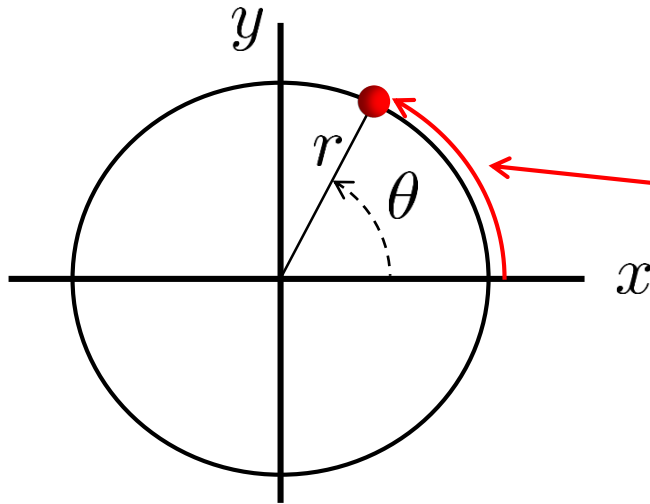
The speed, $v = \text{constant}$

But, the velocity vector, $\vec{v} \neq \text{constant}$

Thus, there must be an acceleration

Angular Position and Velocity

Circular motion can be described using x-y cartesian coordinates, but it's messy. **Angular coordinates** (or polar) work a lot better.



Coordinates: (r, θ)

$s = \text{arc length} = r\theta$

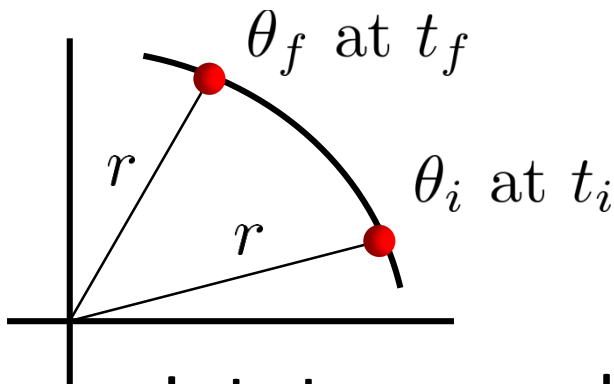
where θ must be in radians

Average angular velocity

between points i and f:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

(Units are radians/second, but watch for your author; he loves different units!)



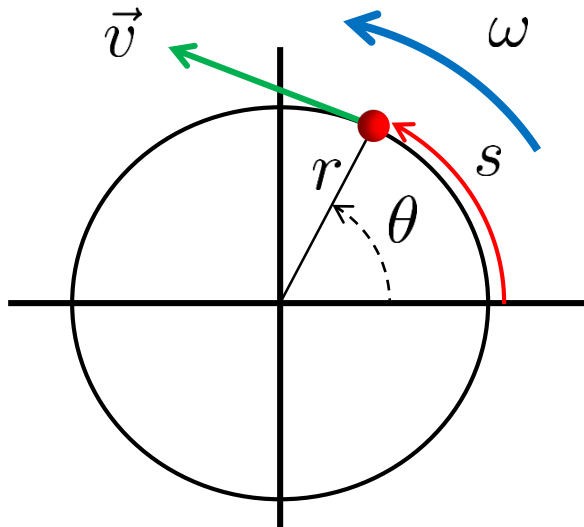
Instantaneous angular velocity: $\omega = \frac{d\theta}{dt}$

Sign Convention: CCW = positive and CW = negative

(CCW = CounterClockWise and CW = ClockWise)

Relations Between Linear and Angular Quantities for UCM

For Uniform Circular Motion (UCM):



$$\text{Arc Length } s = r\theta$$

$$\text{Speed, } v = \frac{ds}{dt} = r \frac{d\theta}{dt} = r\omega$$

Period = time to complete one circle

$$\text{Period, } T = \frac{\text{circumference}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

(see equation sheet)

Whiteboard problem 4-7

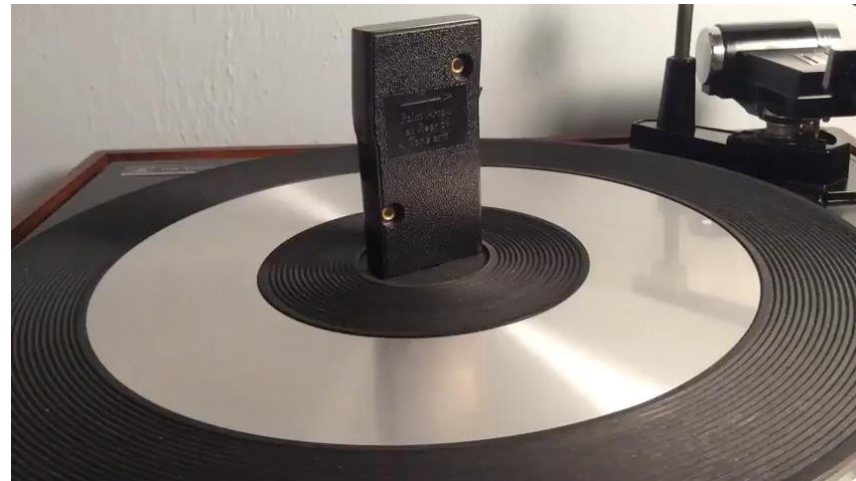
An old fashioned single play vinyl record rotates on a turntable* at 45 rpm (*i.e. revolutions per minute*).

What are

- a) the angular velocity in rad/s? (LC)
- b) the period of the motion? (LC)

**What is a turntable?*

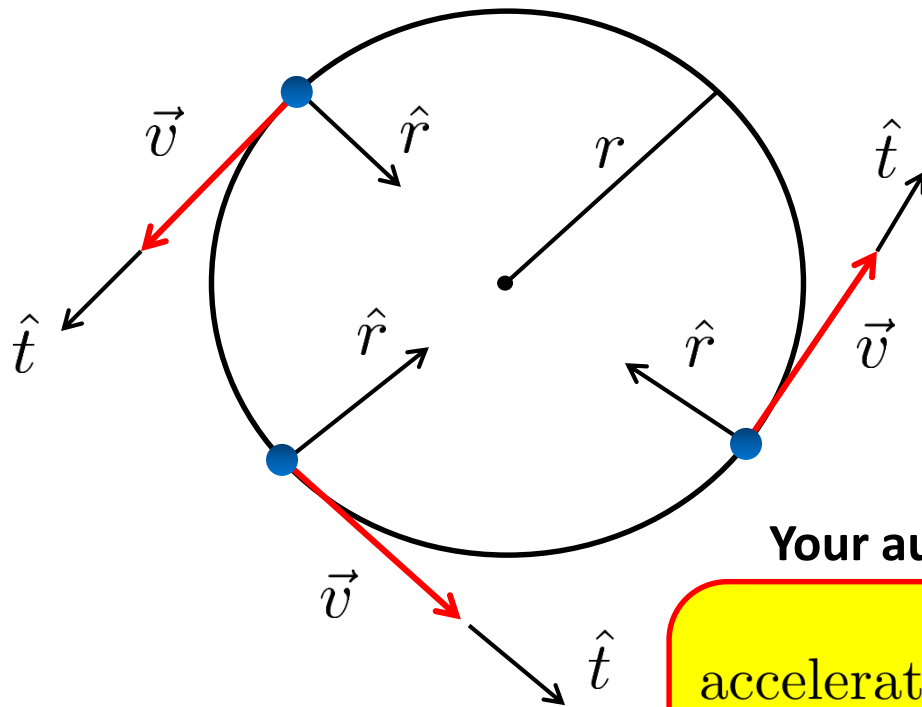
This was our Spotify!



Acceleration for Uniform Circular Motion

(this is the really important part of this section)

For an object in uniform circular motion (UCM), the speed is constant, but the velocity continuously changes direction. Thus, there must be an acceleration.



\hat{r} = The instantaneous radial direction,
Always toward the center of the circle

\hat{t} = The instantaneous tangential direction,
Always tangent to the circle.

For UCM:

\vec{v} is in the tangential direction

\vec{a} is in the radial direction

Your author shows in the text:

acceleration, $\vec{a} = \left(\frac{v^2}{r}, \text{towards the center} \right)$
(or radially inward)

*Valid for the radial component
of acceleration even for
nonuniform circular motion*

“Centripetal”

Whiteboard problem 4-8

The radius of the Earth's very nearly circular orbit around the Sun is 1.5×10^{11} m. **Find:**

- a) The Earth's speed (LC)
- b) The Earth's angular velocity (LC)
- c) The Earth's centripetal acceleration (LC)

How else can you calculate this acceleration? If you know, go ahead and see what you get, and explain to your partners what you're doing.

*Note: to enter an exponential in LC, use e-notation:
i.e. $8.99 \times 10^{-9} = 8.99e-9$*