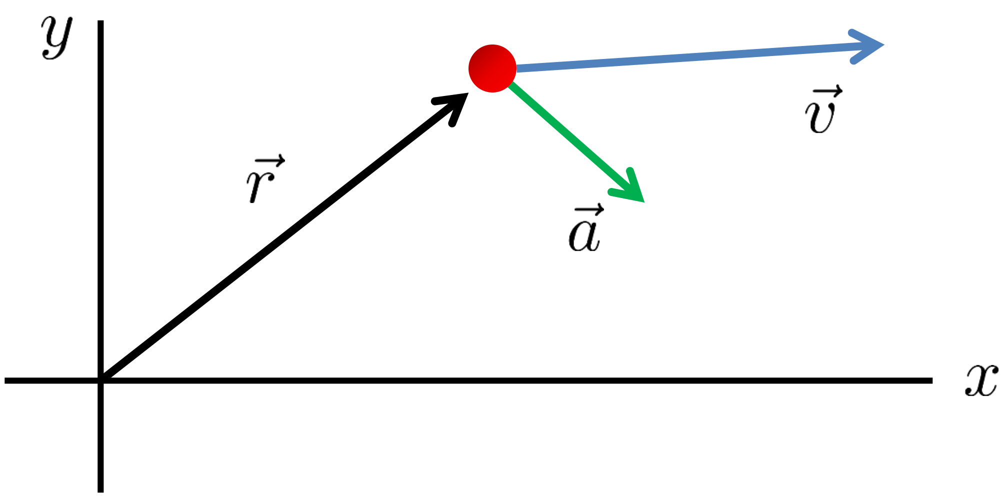
4-1: Two Dimensional Kinematics

Here, we'll extend the 1D kinematics from Chapters 1 & 2 to two dimensions.

So:



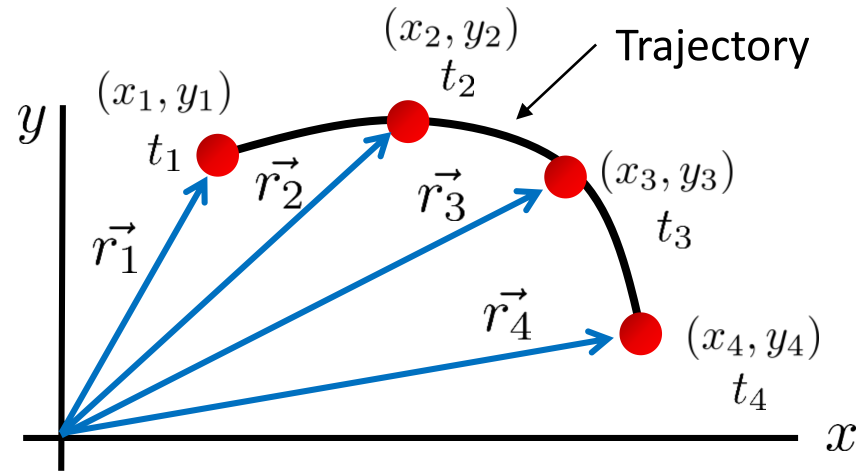
Now, that we know all about Vectors, the Basic Equations are:

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} \qquad \text{and} \qquad \vec{a}(t) = \frac{d\vec{v}(t)}{dt}$$

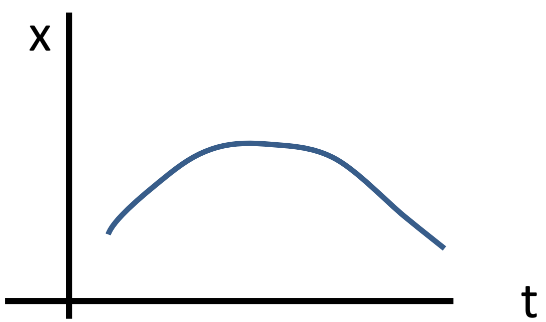
That seems pretty simple, but applying these equations can be difficult. There are some important points to remember.

Point 1: The Trajectory

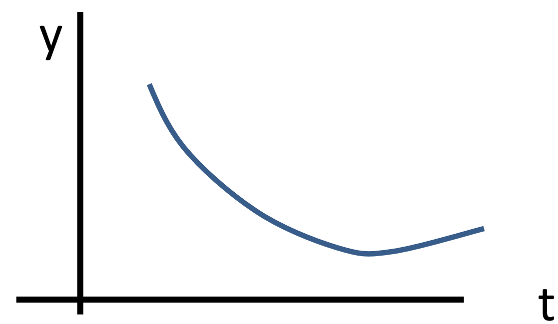
If you plot the (x,y) coordinates of the particle for a series of times or the position vector, $\vec{r}(t)$, you have an actual picture of the motion.
We call this the trajectory of the particle.



Don't confuse the trajectory with plots like these:



and

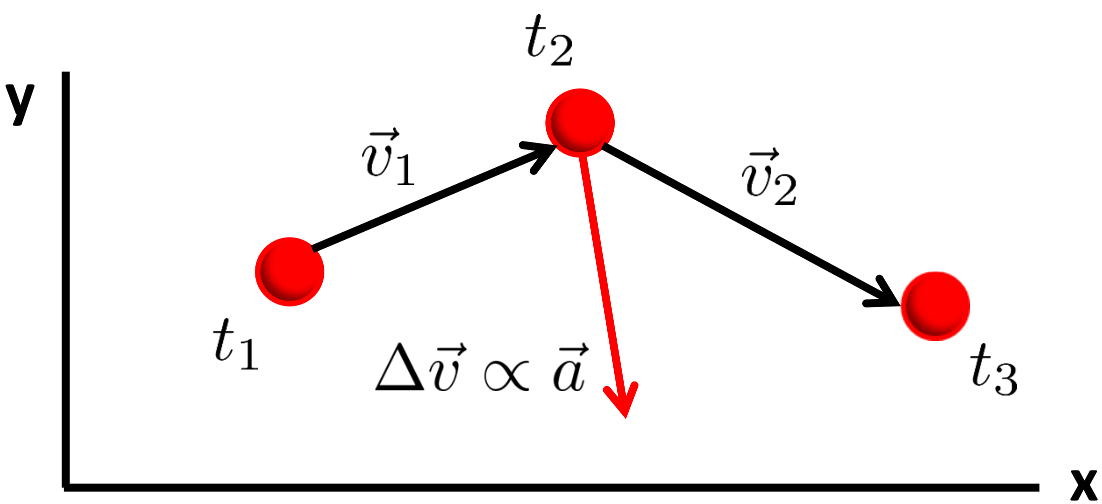


These are **<u>graphs of the functions</u>** that describe mathematically how x and y change with time, not pictures of the motion. We will use both types of plots; don't confuse them.

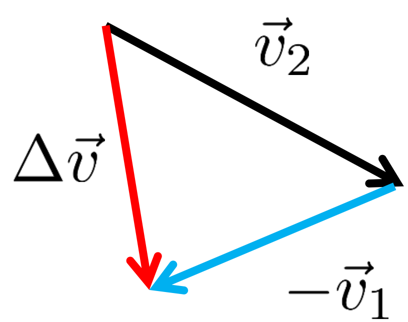
Point 2: Direction of the Velocity

In more than one dimension, the **acceleration can change the magnitude and/or the direction of the velocity**.

Recall the motion diagram from Chapter 1:



$$\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$



How do we solve Problems?

Work with Vector Component Equations! (That's why Chapter 3 is so important!)

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} \quad ; \quad \vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} \quad \& \quad \vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

So the equations: $\vec{v} = \dfrac{d\vec{r}}{dt}$ and $\vec{a} = \dfrac{d\vec{v}}{dt}$ can be written as:

$$v_x\hat{\imath} + v_y\hat{\jmath} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} \quad \text{and} \quad a_x\hat{\imath} + a_y\hat{\jmath} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$$

For two vectors to be equal, their components must be equal.

x-component equations:

$$v_x = \frac{dx}{dt}$$

$$a_x = \frac{dv_x}{dt}$$

y-component equations:

$$v_y = \frac{dy}{dt}$$

$$a_y = \frac{dv_y}{dt}$$

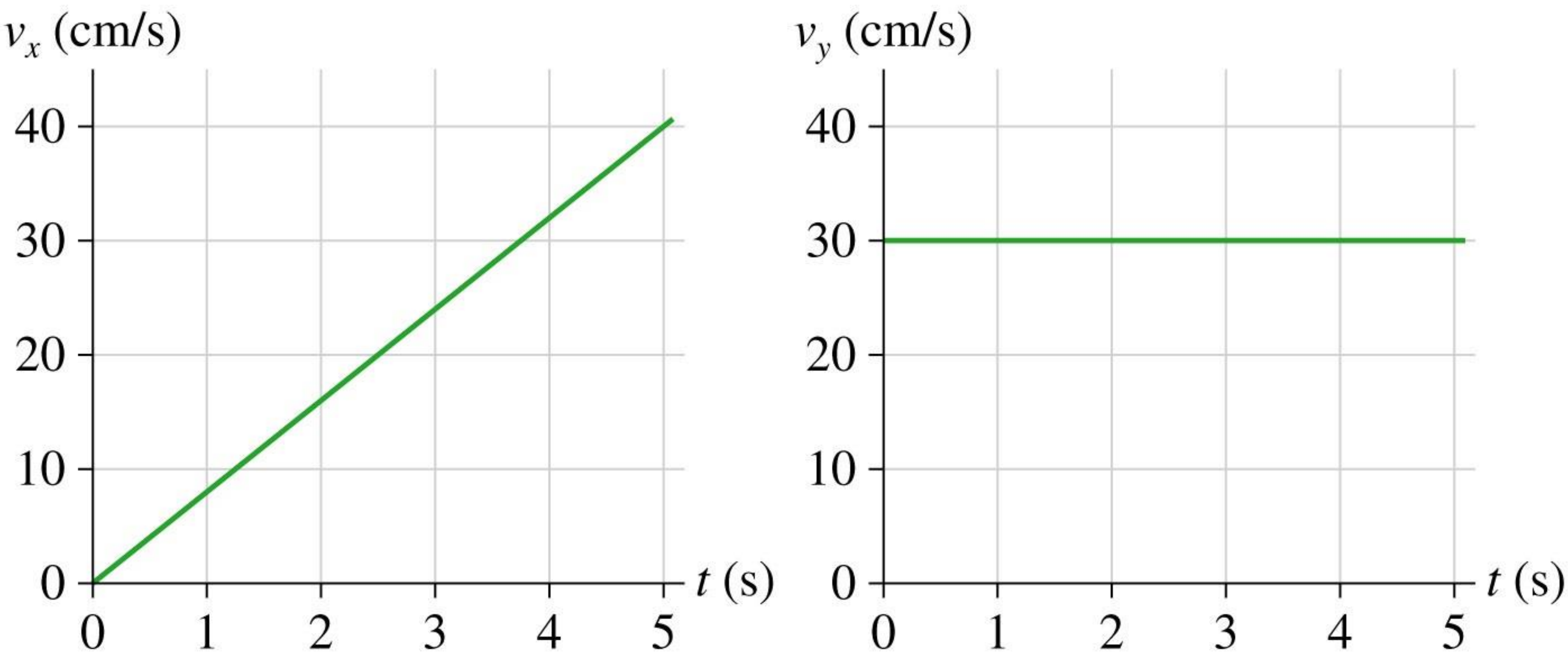
Each component of the motion evolves independently, but they are connected by the time.

Whiteboard Problem 4-1

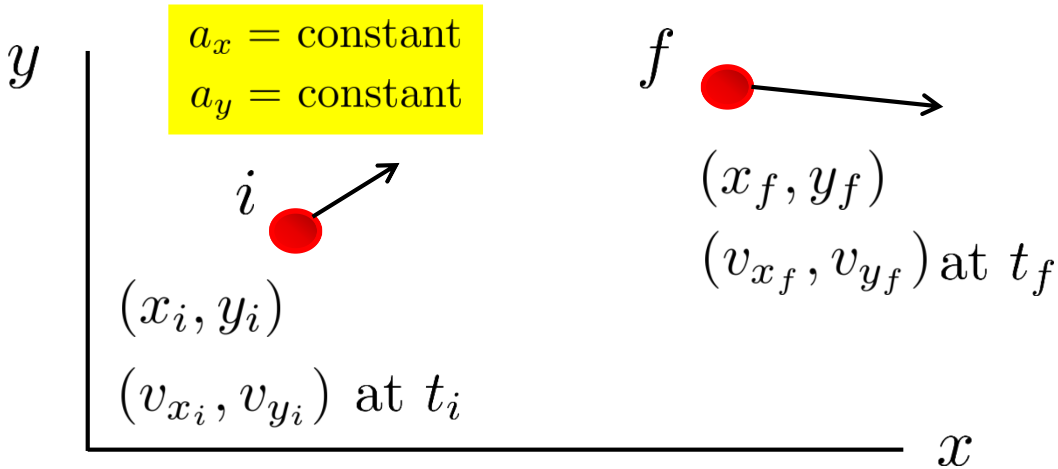
A rocket-powered hockey puck moves on a horizontal frictionless table. The figure below shows graphs of v_x and v_y , the x and y components of the puck's velocity. The puck starts at the origin.

a) **In which direction is the puck moving at t = 2s? Give your answer as an angle CCW from the +x-axis. (LC)**

b. **How far from the origin is the puck at t = 5s? (LC)**



Constant Acceleration



is it v_{i_x} or v_{x_i}?

Knight and I are just going to have to disagree!

For **constant acceleration**, the basic equations integrate to **something familiar**:

x-motion

$$x_f = x_i + v_{x_i}\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$v_{x_f} = v_{x_i} + a_x\Delta t$$

$$v_{x_f}^2 = v_{x_i}^2 + 2a_x\Delta x$$

$$\Delta x = x_f - x_i$$

$$\Delta t = t_f - t_i$$

y-motion

$$y_f = y_i + v_{y_i}\Delta t + \frac{1}{2}a_y(\Delta t)^2$$

$$v_{y_f} = v_{y_i} + a_y\Delta t$$

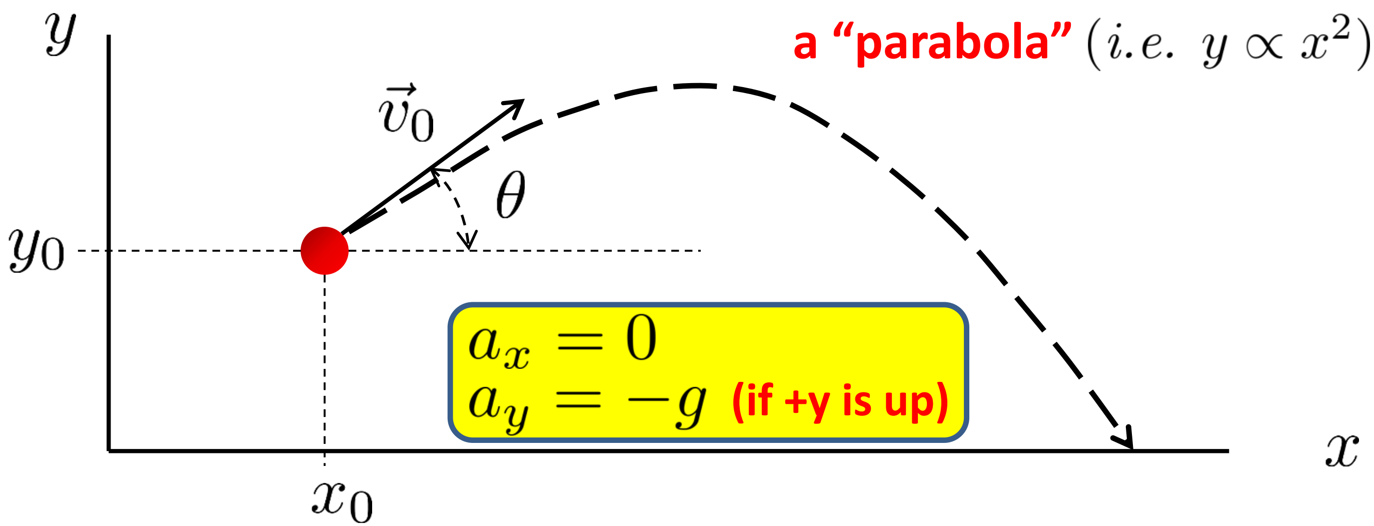
$$v_{y_f}^2 = v_{y_i}^2 + 2a_y\Delta y$$

$$\Delta y = y_f - y_i$$

Do these equations look familiar? They're under Chapter 2 on the equation sheet, but now we have one set for the x-motion and one for the y-motion. But nothing new here; we know how to work with these equations.

Projectile Motion

-A Special Case of 2D Constant Acceleration



Just use the above 2D constant acceleration equations with:

$$v_{x_0} = v_0 \cos\theta \qquad a_x = 0$$

and

$$v_{y_0} = v_0 \sin\theta \qquad a_y = -g \text{ (if +y is up)}$$

Important Note: this all assumes that we have chosen +y to be up!

(Note velocity and acceleration components; you'll use this simulation in HW) PhET

Whiteboard Problem 4-2

A rifle is aimed horizontally at a target 50 m away. The bullet hits the target 2.0 cm below the aim point.

a) **Draw the Problem on your Whiteboard.**
b) **What is the bullet's flight time? (LC)**
c) **What was the bullet's speed as it left the barrel? (LC)**

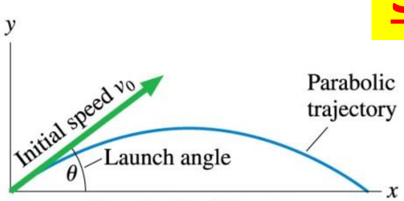
Some Good Stuff from the Text

MODEL 4.1

Projectile motion

For motion under the influence of only gravity.

- Model the object as a particle launched with speed v_0 at angle θ:



A projectile follows a parabolic trajectory.

- Mathematically:
  - **Uniform motion** in the horizontal direction with $v_x = v_0 \cos\theta$.
  - **Constant acceleration** in the vertical direction with $a_y = -g$.
  - Same Δt for both motions.
- Limitations: Model fails if air resistance is significant.

PROBLEM-SOLVING STRATEGY 4.1 MP

Projectile motion problems

MODEL Is it reasonable to ignore air resistance? If so, use the projectile motion model. **(Draw the Problem!)**

VISUALIZE Establish a coordinate system with the x-axis horizontal and the y-axis vertical. Define symbols and identify what the problem is trying to find. For a launch at angle θ, the initial velocity components are $v_{ix} = v_0 \cos\theta$ and $v_{iy} = v_0 \sin\theta$.

SOLVE The acceleration is known: $a_x = 0$ and $a_y = -g$. Thus the problem is one of two-dimensional kinematics. The kinematic equations are

Horizontal	Vertical
$x_f = x_i + v_{ix}\,\Delta t$	$y_f = y_i + v_{iy}\,\Delta t - \frac{1}{2}g(\Delta t)^2$
$v_{fx} = v_{ix} = \text{constant}$	$v_{fy} = v_{iy} - g\,\Delta t$

Δt is the same for the horizontal and vertical components of the motion. Find Δt from one component, then use that value for the other component. *

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

Show important points in the motion. →

Assuming +y is up. →

Not new equations, constant acceleration equations from chapter 2, don't forget the v^2 equation for the y-motion. →

$$v_{f_y}^2 = v_{i_y}^2 - 2g\Delta y$$

* This is what we did in the last WB problem.

Whiteboard Problem: 4-3

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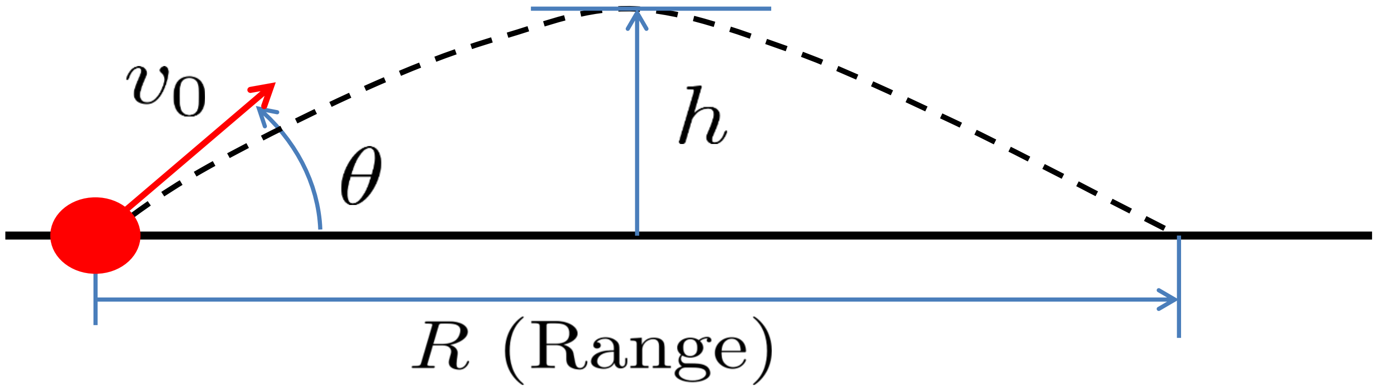
A projectile is launched from the ground with an initial speed v_0 at an angle θ from the horizontal and returns to the ground.



a) **Draw it** and find an expression for the projectile's maximum height, h, in terms of v_0, θ, and constants. **(LC, use $(v_0)^2$ for v_0^2)**

b) **Draw it** and find an expression for the projectile's range, R, *i.e.* the horizontal distance it travels before it hits the ground. Your expression should be in terms of v_0, θ, and constants.

(LC, use $(v_0)^2$ for v_0^2)

c) What angle, θ, gives the maximum range?

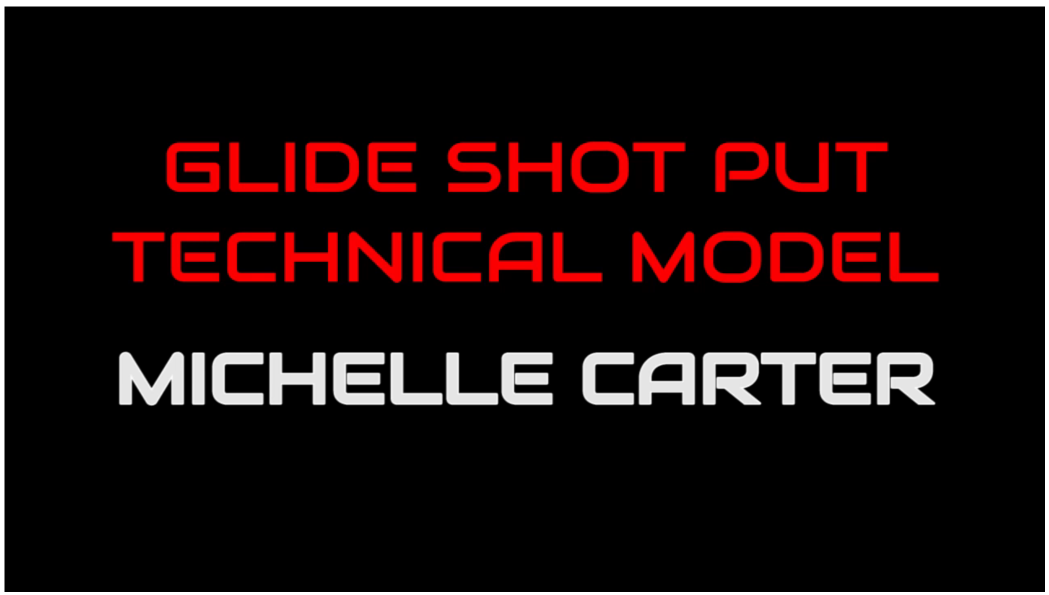
Whiteboard Problem 4-4

In the Olympic shotput event, an athlete throws the shot with an initial speed of 12.0 m/s at a 40^o angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground.

a) Draw the Problem

b) How far does the shot travel (*in the horizontal direction*)? (LC)

PhET



Michelle Carter wins the Olympic Shot Put in 2016
(she repeated in 2020, actually 2021)