Vectors

In Chapter 1, we introduced the concept of a **Vector**:

A Vector is a quantity that has both magnitude and direction.

Some examples of vector quantities that we'll see in **PHY181** are position $\vec{r}$, velocity $\vec{v}$, acceleration $\vec{a}$, force $\vec{F}$, momentum $\vec{p}$, and many others.

Quantities that have only a magnitude, like mass, m, Temperature, T, are called **Scalars**.

In Chapter 3, we want to develop and learn how to work with vectors analytically. **In what we're going to do in PHY181 and PHY182,**
How important is this in Physics and Engineering?

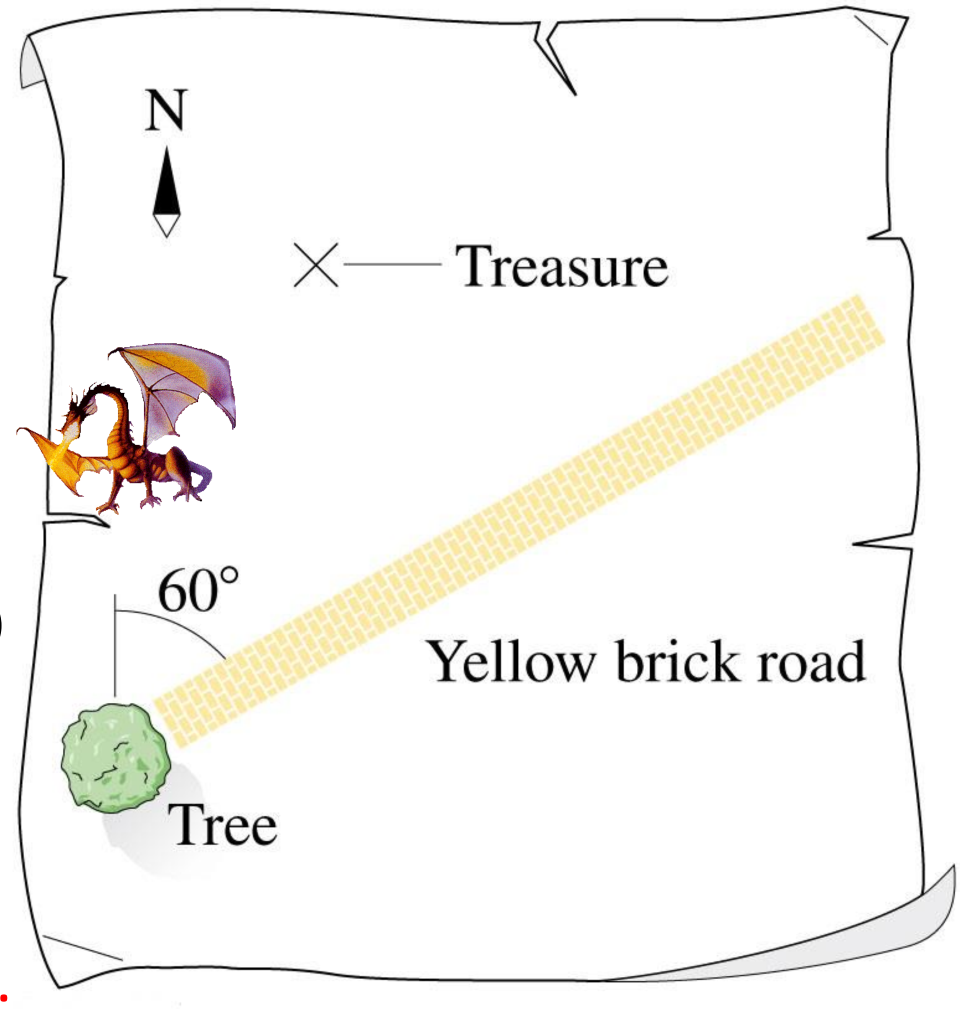
$$(\textbf{Extremely})^n \quad \textbf{where } n >> 1$$

$$\text{Or even } (\textbf{Extremely})^{n!} \quad \textbf{where } n >> 1$$

If you are an engineering student, how many times will you be taught the basics of vectors?

Whiteboard: Problem 3-1

The treasure map shown here gives the following directions to the buried treasure: ***"Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig."*** But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle of 60° east of north. After walking 300 paces, you see an opening through the woods. **Which direction should you go (i.e. compass heading), and how far (i.e. number of paces) to reach the treasure?**

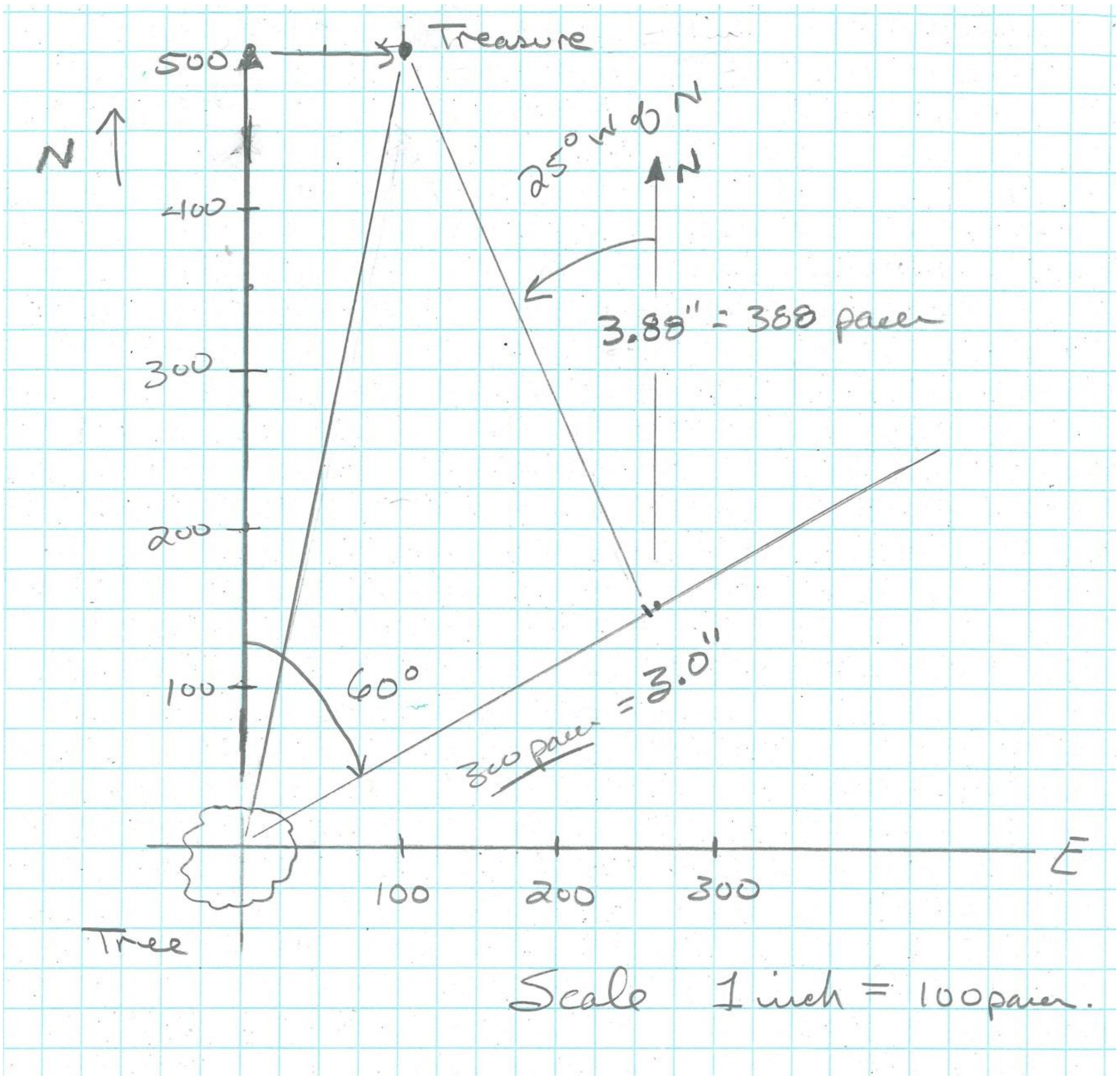


Here's a problem you can solve without the use of vectors. **Solve this problem graphically <u>using only the graph paper, rulers, and protractors provided</u>.**

I would suggest a scale of 1 inch = 100 paces.

Enter your <u>number of paces</u> on LC

Here's my solution: 388 paces at 25^o West of North



After we learn how to work with vectors, we'll see that this is an easy problem, and we can get a much more accurate answer.

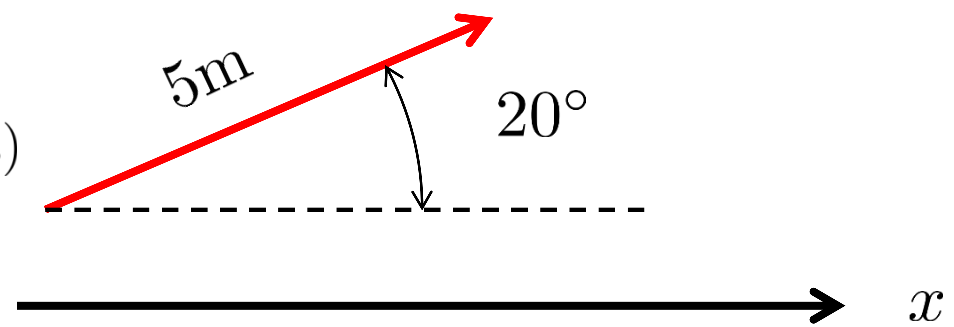
Vectors: The Very Basics . . . Arrows

One way that Knight denotes a vector:

$$\vec{A} = \text{(length or } \underline{\text{magnitude}}\text{, direction)}$$

Note: the magnitude of a vector is always greater than or equal to zero. **It is never negative!**

e.g. $\vec{A} = (5\text{m}, 20^\circ \text{ above } +x \text{ axis})$



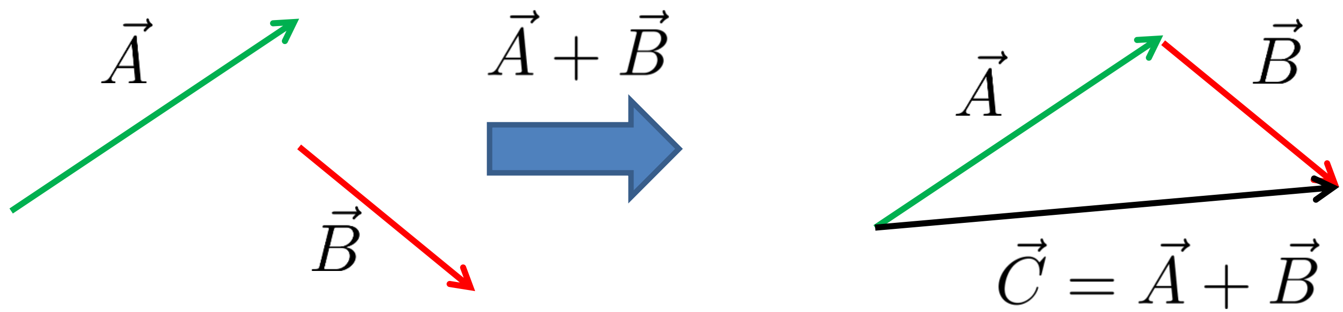
Important Notation Convention:

- **A quantity with an arrow above it, is a vector; a quantity without an arrow is a scalar.**
- So: $\vec{A}$ denotes the vector, but A (or $|\vec{A}|$) denotes the magnitude of the vector and is a non–negative scalar.

Vector Addition and Subtraction . . . With Arrows

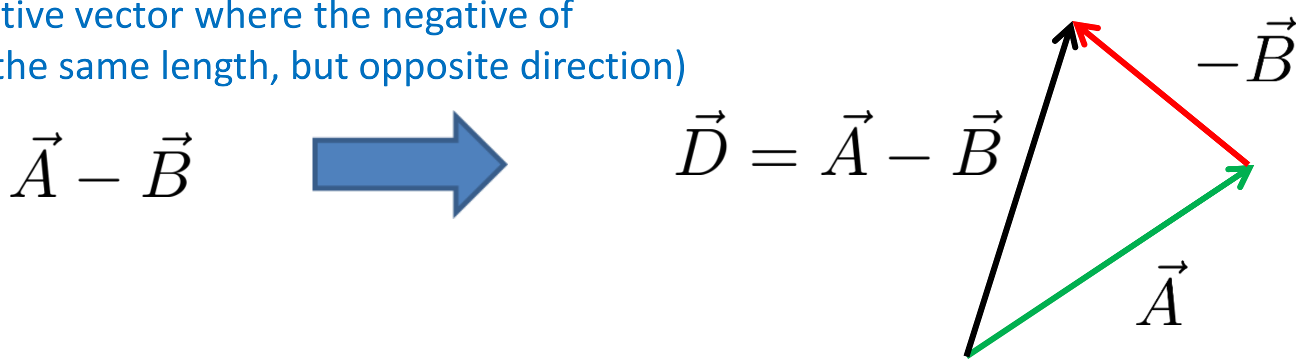
In Chapter 1, we saw how to add or subtract vectors graphically.
This is useful, but cumbersome and not very accurate. **Here's a review:**

Vector Addition: move the vectors, don't rotate or stretch; place tails on heads.



Vector Subtraction: treat as: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$

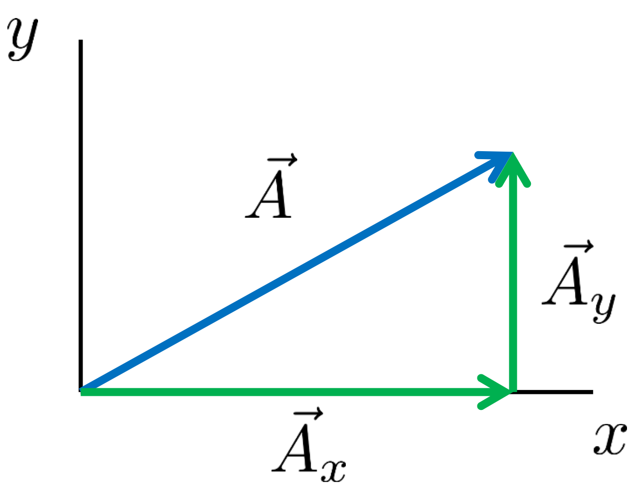
(add the negative vector where the negative of
a vector has the same length, but opposite direction)



Component Vectors & Vector Components ????

Your author kind of confuses things here by defining *"component vectors"* (which are never used again) and ***"vector components"*** (which will be used forever). **The two are not the same thing; however, they are related.**

Component Vectors: (with respect to some coordinate system)



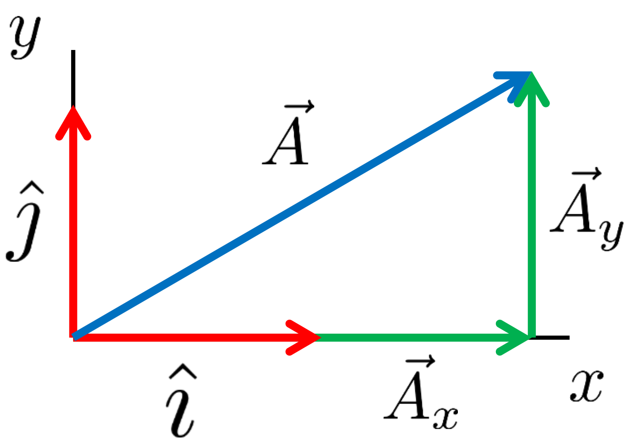
From vector addition:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$\vec{A}_x$ and $\vec{A}_y$ Are called the **"Component Vectors"** of vector $\vec{A}$

Vector Components (the useful one)

Coordinate Unit Vectors:



Define the Unit Vectors:

what does unit mean?

magnitude = 1

$$\hat{\imath} \equiv (1, +x \text{ direction})$$

$$\hat{\jmath} \equiv (1, +y \text{ direction})$$

(Note: unit vectors wear hats)

So, using the "Component Vectors" from above:

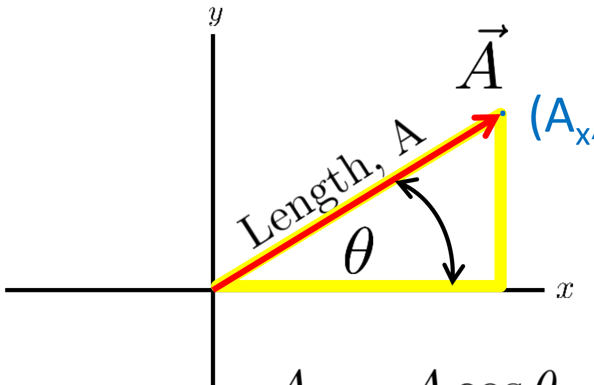
$$\vec{A} = \vec{A}_x + \vec{A}_y \equiv A_x\hat{\imath} + A_y\hat{\jmath}$$

where: A_x and A_y are called **The Components of Vector $\vec{A}$**

Note: the **Components of a Vector** are scalars and can be positive, negative, or zero. In a few of the problems that we'll do, we'll need three dimensions, i.e. a z-axis too; the unit vector in the z-direction is $\hat{k}$

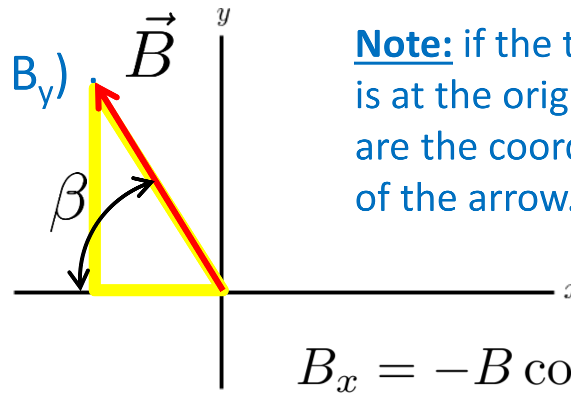
Finding Vector Components . . . *Really Important Stuff*

Suppose that we know the magnitude and direction of the vector and we want the components: i.e. we know: $\vec{A} = (A, \theta \text{ relative to some direction})$



$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

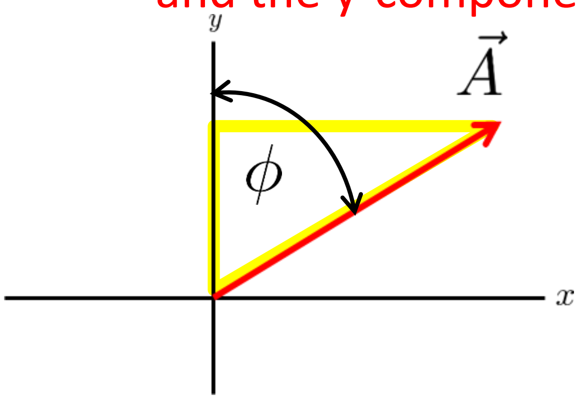
Or



Note: if the tail of the vector is at the origin, the components are the coordinates of the tip of the arrow.

$$B_x = -B\cos\beta$$
$$B_y = B\sin\beta$$

Note: a common mistake is to always associate the x-component with the cosine and the y-component with the sine. What about:



Here:

$$A_x = A\sin\phi$$
$$A_y = A\cos\phi$$

You have to look at the triangle and the axes!

Whiteboard Problem: 3-2

Draw each of the following vectors, then **find its x and y components, and write the vector in component form.**

a) $\vec{r} = (100 \text{ m}, 45^\circ \text{ below the positive x-axis})$ **(LC)**

b) $\vec{v} = (300 \text{ m/s}, 20^\circ \text{ above the positive x-axis})$ **(LC)**

c) $\vec{a} = (5.0\ m/s^2, \text{ negative y direction})$ **(LC)**

Instructions for entering a vector in component form on LC: For the vector:

$$\vec{E} = -1.523\hat{\imath} + 3.736\hat{\jmath}$$

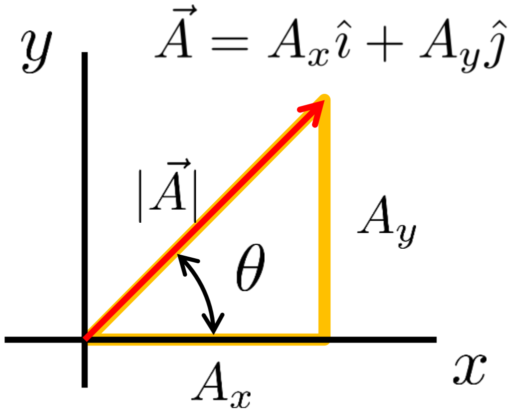
MP is much better with Vectors.

Use only three significant figures, put the components in parentheses, and use just i and j for unit vectors – no hats! For the vector above, enter:

(-1.52)i + (3.74)j

How About Going the Other Way?

Suppose we know a vector's components, how do we find its magnitude and direction?



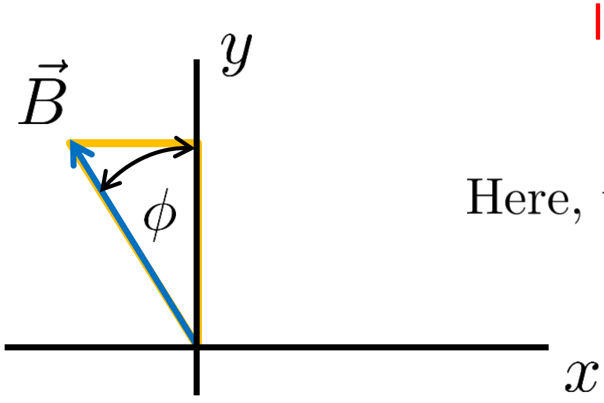
From the Pythagorean theorem:

$$\text{Magnitude of } \vec{A} = A = |\vec{A}| = \sqrt{{A_x}^2 + {A_y}^2}$$

(Note: Both A and $|\vec{A}|$ denote the Vector Magnitude)

$$\text{Direction: } \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Be careful with the direction: you have to be smarter than your calculator; look at the triangle, e.g:



$$\text{Here, the angle from } +y \text{ axis: } \phi = \tan^{-1}\left(\frac{|B_x|}{B_y}\right)$$

Again, you have to look at the triangle.
It's always best to draw it!

Whiteboard Problem: 3-3

Draw each of the following vectors, label an angle that specifies the vector's direction, **then find the vector's magnitude and direction.**

a) $\vec{A} = 4\hat{\imath} - 6\hat{\jmath}$ **LC: Enter Magnitude**

b) $\vec{r} = 50\hat{\imath} + 80\hat{\jmath}\ m$ **LC: Enter angle CCW* from +x axis**

c) $\vec{v} = -20\hat{\imath} + 40\hat{\jmath}\ m/s$ **LC: Enter Magnitude**

d) $\vec{a} = 2.0\hat{\imath} - 6.0\hat{\jmath}\ m/s^2$

LC: Enter angle CCW* from +x axis

***CCW = Counter Clock Wise**

Adding and Subtracting Vectors with Components

To add two or more vectors, you just add up their like components:

e.g.
$$\vec{C} = \vec{A} + \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})$$
$$= \underbrace{(A_x + B_x)}_{= C_x}\hat{\imath} + \underbrace{(A_y + B_y)}_{= C_y}\hat{\jmath}$$

Likewise, to subtract two vectors, you just subtract their like components:

e.g.
$$\vec{D} = \vec{A} - \vec{B} = \underbrace{(A_x - B_x)}_{= D_x}\hat{\imath} + \underbrace{(A_y - B_y)}_{= D_y}\hat{\jmath}$$

If you know the components of a vector, you know everything about it.

Working with vectors in terms of their components is incredibly useful. Later, when we need them, we'll also develop the dot product and the cross product.

Whiteboard Problem: 3-4

Let $\vec{A} = 4\hat{\imath} - 2\hat{\jmath}$, $\vec{B} = -3\hat{\imath} + 5\hat{\jmath}$, and $\vec{F} = \vec{A} - 4\vec{B}$.

a) Determine the vector $\vec{F}$ in component form. **(LC)**

b) Draw a coordinate system and on it show the vectors $\vec{A}$, $\vec{B}$, and $\vec{F}$. (Try to get the scale right.)

c) What are the magnitude and direction of the vector $\vec{F}$? **(Enter the magnitude in LC)**

Instructions for entering a vector in component form on LC: For the vector:

$$\vec{E} = -1.523\hat{\imath} + 3.736\hat{\jmath}$$

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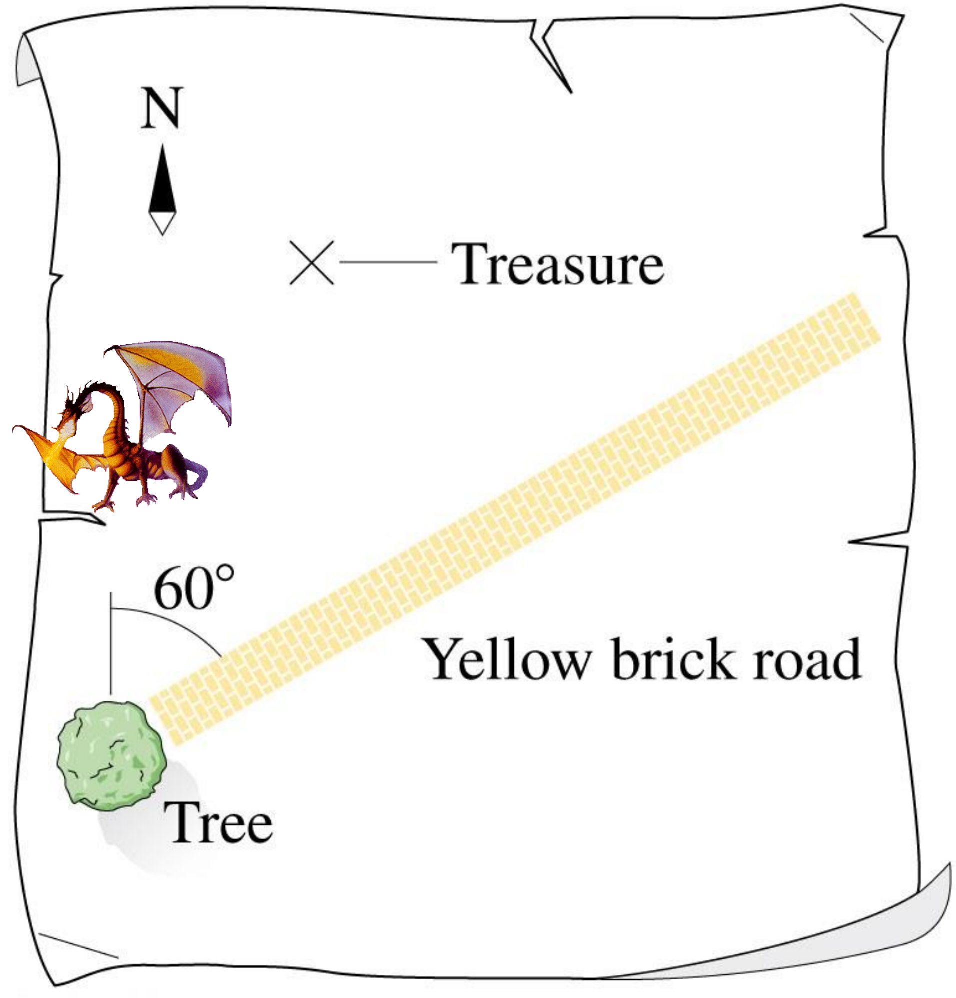
Whiteboard Problem 3-5 (WB 3.1 Re-visited)

The treasure map shown here gives the following directions to the buried treasure: ***"Start at the old oak tree, walk due north for 500 paces, then due east for 100 paces. Dig."*** But when you arrive, you find an angry dragon just north of the tree. To avoid the dragon, you set off along the yellow brick road at an angle of 60° east of north. After walking 300 paces, you see an opening through the woods. **Which direction should you go (i.e. compass heading), and how far (i.e. number of paces) to reach the treasure?**



Now solve this using vector components. Make sure to make a sketch and define a coordinate system. But your sketch doesn't have to be to scale. Define vectors in terms of their components.

How does your answer compare with the graphical solution?

Enter your number of paces (*to within 0.1 pace*) on LC