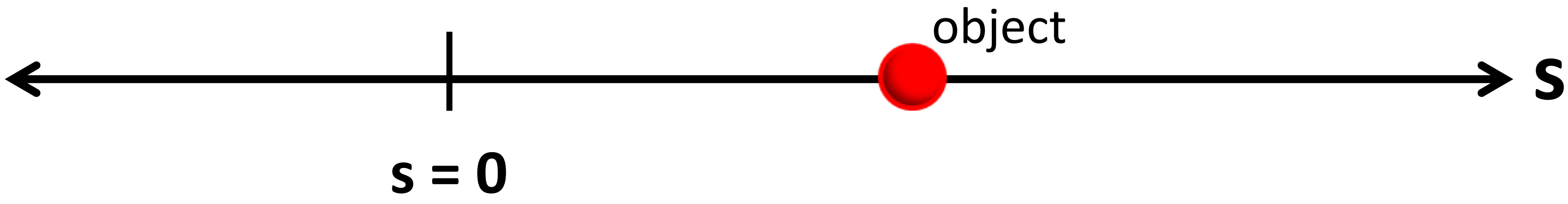
Motion in One Dimension*

Motion along the s-axis (*why s? s here stands for either y or x*):



s = object's **position** coordinate along the s-axis (can be +, -, or 0)

$\mathbf{v_s}$ = object's **velocity** along the s-axis (can be +, -, or 0)

$\mathbf{a_s}$ = object's **acceleration** along the s-axis (can be +, -, or 0)

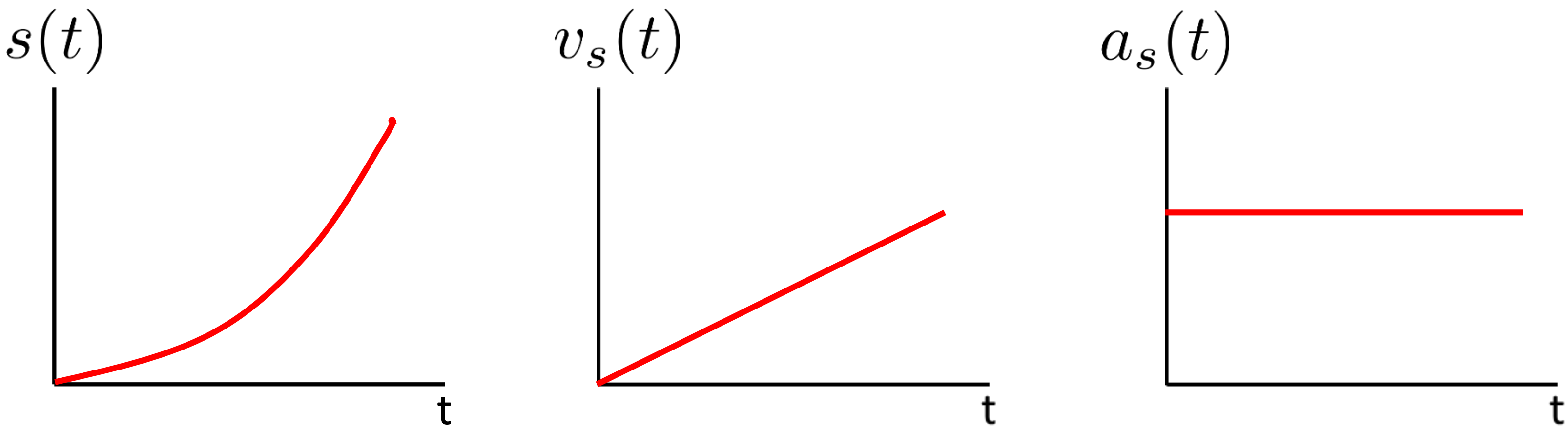
*Note, in **one dimension**, we can drop the vector notation. Looking ahead, these are the s-components of the vectors $\vec{r}$, $\vec{v}$, and $\vec{a}$

Motion in One Dimension

In Chapter 1, we introduced motion diagrams to describe motion. **Now, we want to represent the motion as mathematical functions of time**

$$s(t),\ v_s(t),\ \text{and}\ a_s(t)$$

e.g. we may have something like this, i.e. graphs of the functions:

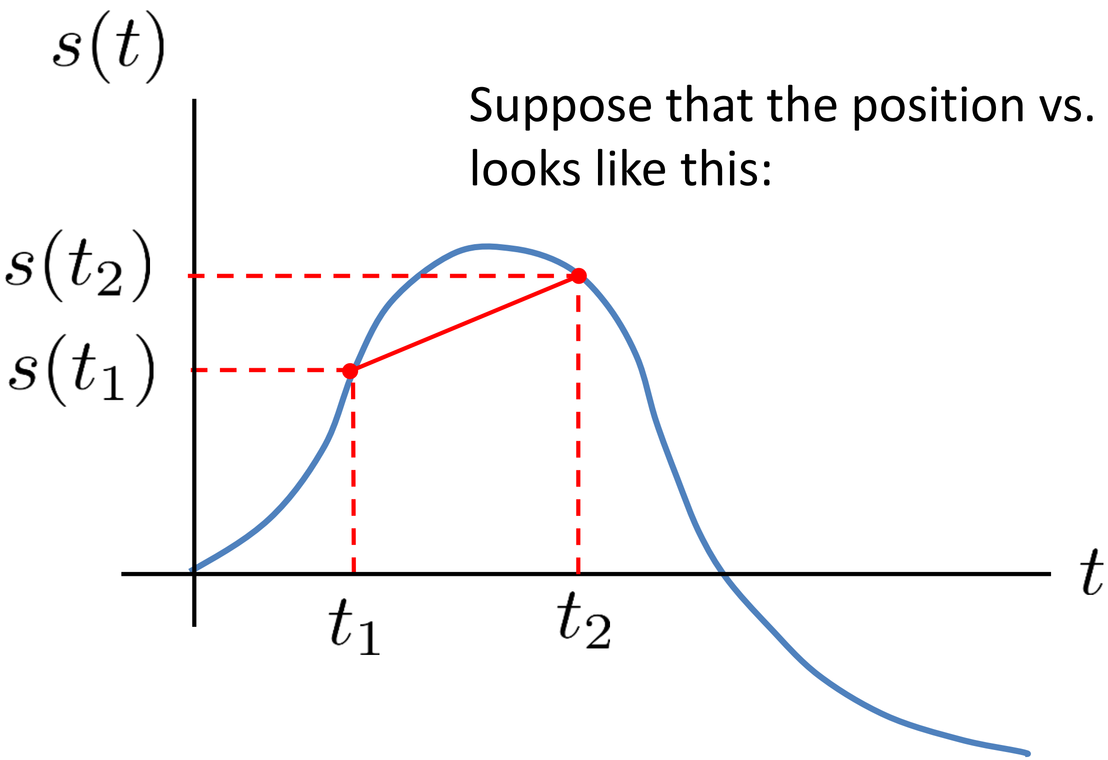


Note, these are graphs – not pictures. The object is still moving in one dimension along the s-axis. **Our goal in Chapter 2 is to develop the mathematical relations between s(t), v_s(t), and a_s(t).**

General 1D Motion: Average Velocity

$(a_s = a_s(t) \neq \text{constant})$

Suppose that the position vs. time looks like this:



Average Velocity between t_1 & $t_2 = v_{s_{\text{avg}}} = \dfrac{\Delta s}{\Delta t} = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

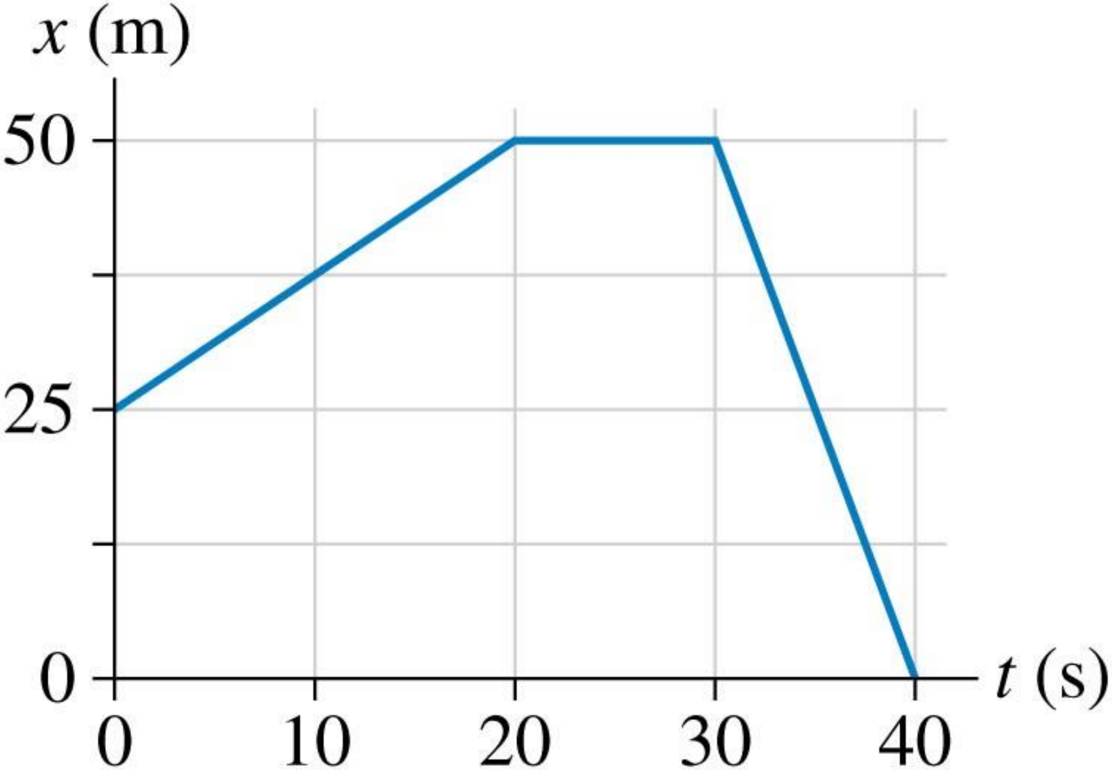
= slope of the line connecting points 1 & 2

Whiteboard Problem: 2-1

The figure below is the position vs. time graph of a jogger.

What is the jogger's velocity at

a) **t = 10s (LC)**
b) **t = 25s**
c) **t = 35 s (LC)**



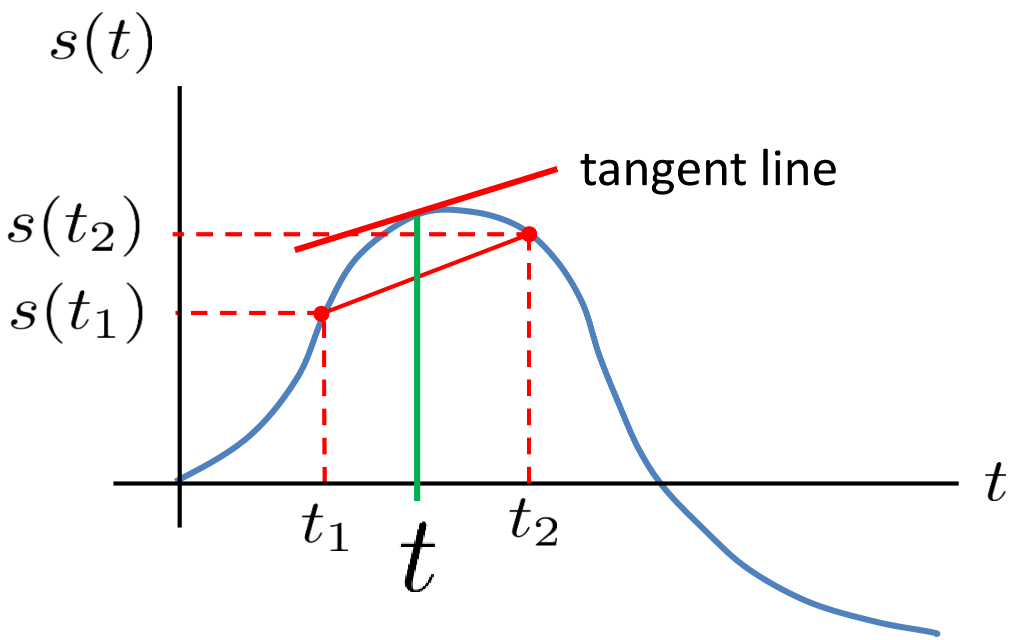
General 1D Motion: Instantaneous Velocity

$$(a_s = a_s(t) \neq \text{constant})$$

So, we have:

Average Velocity between $t_1 \ \& \ t_2 = v_{s_{\text{avg}}} = \dfrac{\Delta s}{\Delta t} = \dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$

= slope of the line connecting points 1 & 2



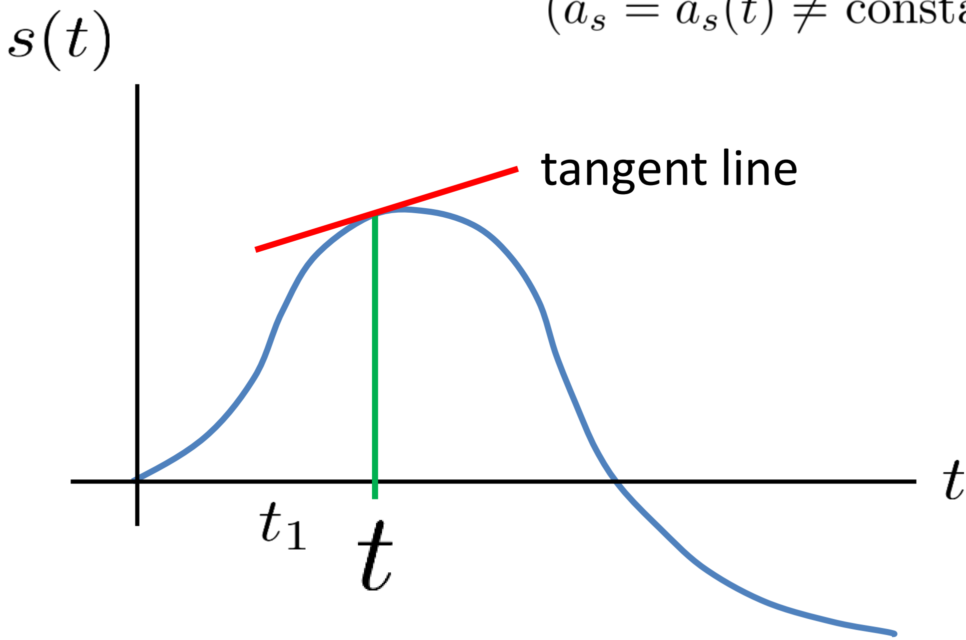
How do we find the velocity at at a given instant of time t?

We let t_1 and t_2 get closer and closer together until they're infinitesimally close to t.

The line connecting t_1 and t_2 when they are infinitesimally close to t is the tangent line.

General 1D Motion: Instantaneous Velocity

$(a_s = a_s(t) \neq \text{constant})$



Instantaneous velocity at t, $v_s(t) = \dfrac{ds}{dt}$ = slope of the tangent line at t.

= instantaneous rate of change of s(t)

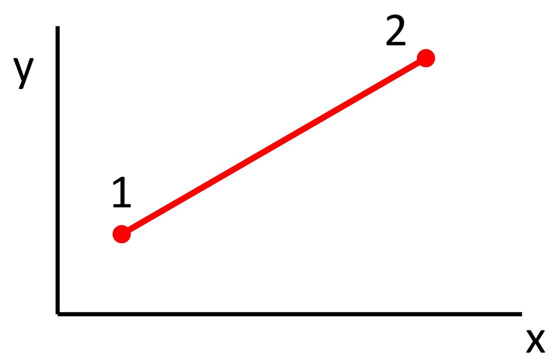
= **<u>derivative</u> of s(t) evaluated at t**

Note: ds and dt are Δs and Δt when the points are infinitesimally close.

What about Derivatives?

Derivatives (from Calculus*)

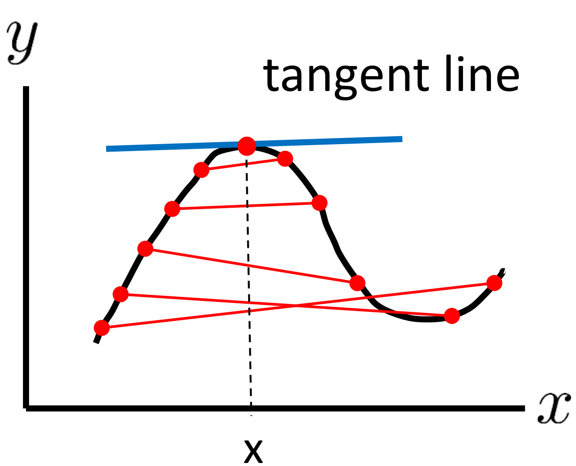
Slope of a straight line:



Slope for any point on the line $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

Derivative = constant = slope between 1 and 2.

Curve: what is the slope at x?



As the two points used for the slope get closer together $\frac{\Delta y}{\Delta x} \rightarrow \frac{dy}{dx}$

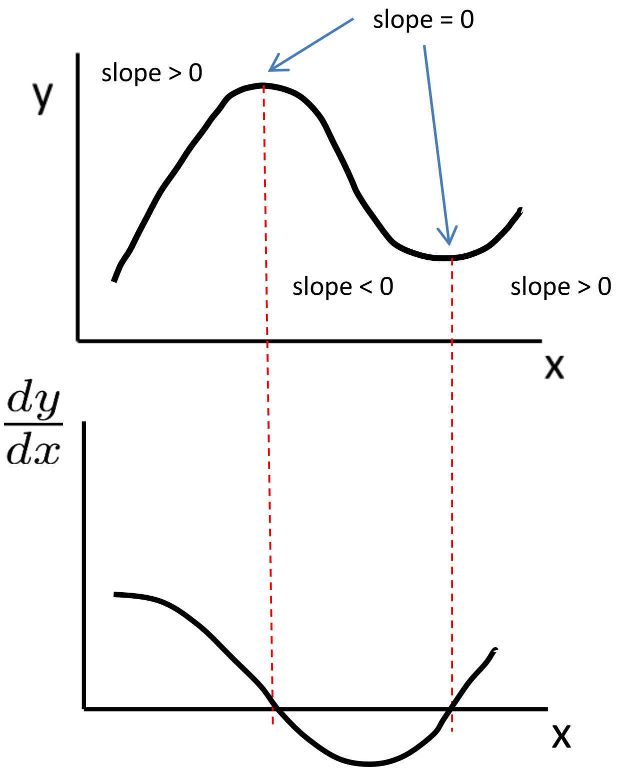
$\frac{dy}{dx}$ is called the **derivative** of the function y

It gives the slope of the tangent line at the point x, or the instantaneous rate of change of the function at the point x.

*If you haven't seen this in Calculus yet – you will very soon. There, you'll do all the theorems, proofs, etc. Here, we just need to understand what a derivative is and how to calculate some very simple ones.

What kinds of derivatives will we need to do?

Graphical:



Analytical:

Horizontal straight line:

$$y(x) = \text{constant} \qquad \frac{dy}{dx} = 0$$

Straight line with slope m:

$$y(x) = mx + b \qquad \frac{dy}{dx} = m$$

Power law function:

$$y(x) = cx^n \qquad \frac{dy}{dx} = cnx^{n-1}$$

$(c \;\&\; n \text{ are constants})$

Sum of two functions:

$$\frac{d}{dx}[f(x) + g(x)] = \frac{df(x)}{dx} + \frac{dg(x)}{dx}$$

We'll have other rules as we need them.

Whiteboard Problem 2-2

Determine expressions for the derivatives of the following functions.* **(Enter your expressions on LC)**

$$\text{a) } f(x) = 10x^3$$

$$(\text{Answer: } 30x^2)$$

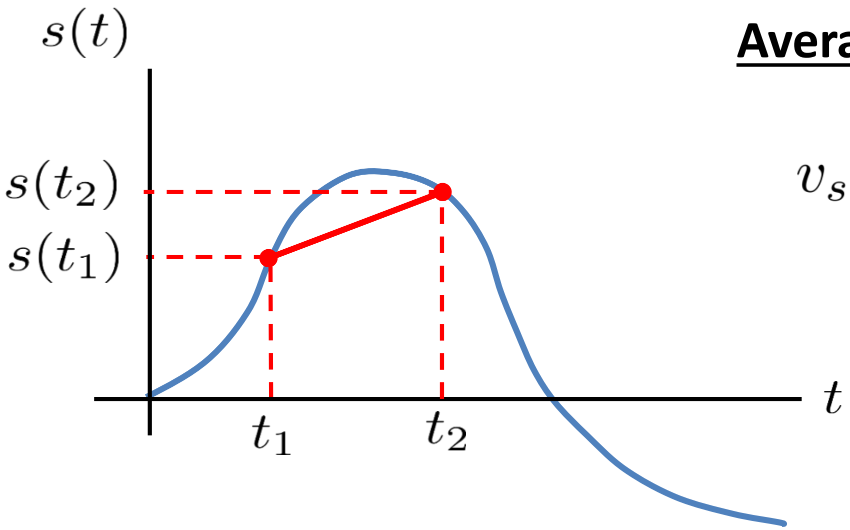
$$\text{b) } f(x) = \frac{6}{x} + 5x$$

$$(\text{Answer: } -\frac{6}{x^2} + 5)$$

Now, a quick review for what this means for **velocity**:

**If you have seen this before in Calculus, show all of the steps on your WB, and help your group members who might be taking Calculus I now.*

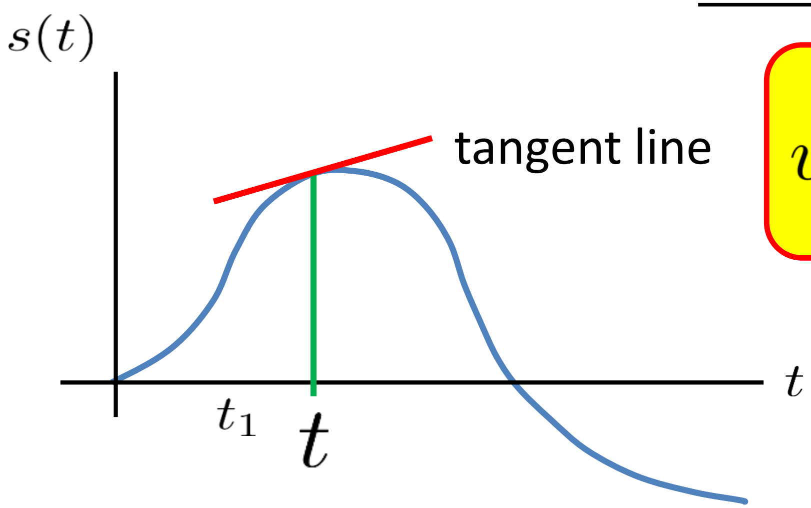
A Quick Review of What We Have for Velocity



Average Velocity Between t_1 and t_2:

$$v_{s_{\text{avg}}} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

= slope of the line connecting points 1 & 2



Instantaneous Velocity At Some Time t:

$$v_s(t) = \frac{ds}{dt}$$

= derivative of s(t) evaluated at t

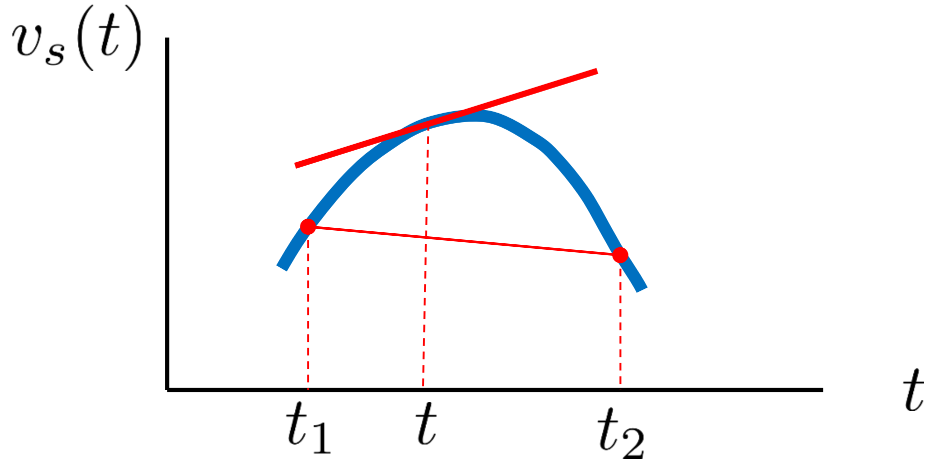
= slope of the tangent line at t.

= instantaneous rate of change of s(t)

Acceleration is the Rate of Change of Velocity

Now that we know a little about slopes and derivatives, the acceleration is obtained from the velocity in a similar manner.

Suppose that the velocity vs. time looks like this:



Average Acceleration between t_1 and t_2 $= a_{s_{\text{avg}}} = \dfrac{\Delta v_s}{\Delta t} = \dfrac{v_s(t_2) - v_s(t_1)}{t_2 - t_1}$

Instantaneous Acceleration at t $= a_s(t) = \dfrac{dv_s}{dt}$

= derivative of v(t) evaluated at t

= slope of the tangent line at time t

= instantaneous rate of change of v_s(t)

Whiteboard Problem: 2-3

A particle moving along the x-axis has its position described by

$$x = (2t^2 - t + 1) \text{ meters}$$

where t is in seconds.

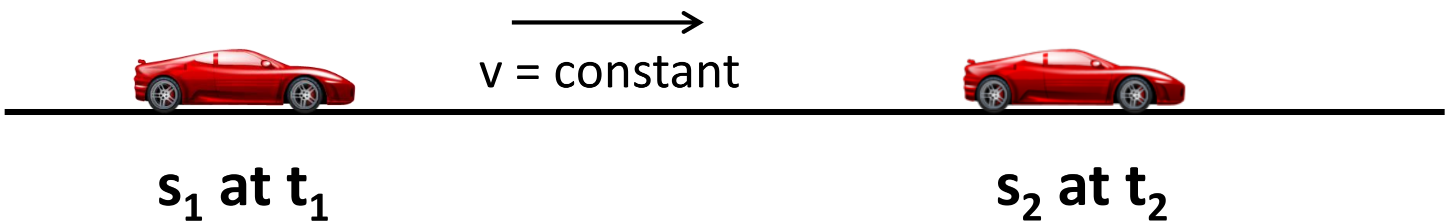
At t = 2s, what are the particle's

a) **position in m (LC)**
b) **velocity in m/s (LC)**
c) **acceleration in m/s² (LC)**

How to get Position from Velocity

We've seen how to get velocity from position: $v_s(t) = \frac{ds}{dt}$

How do we get position from velocity? This is something you do everyday, *e.g. suppose you're driving in your car at constant velocity:*



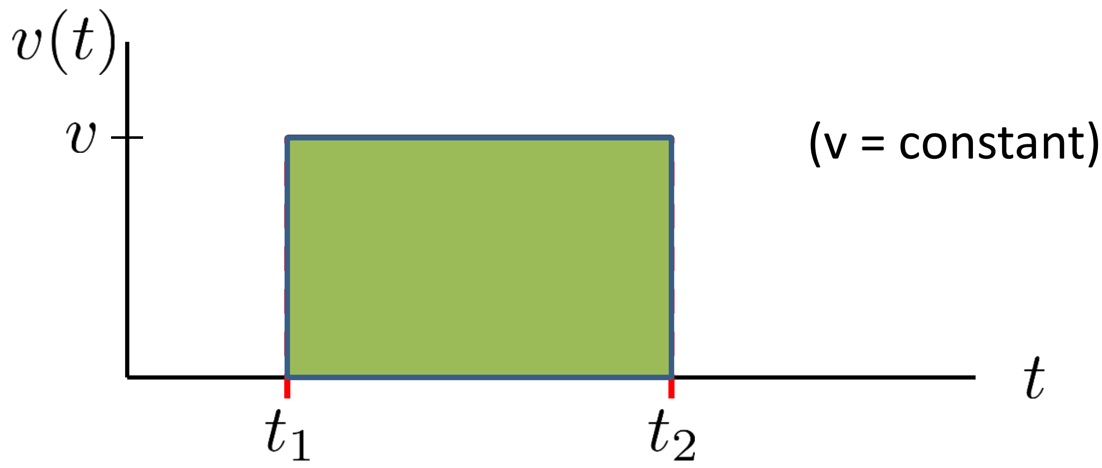
At time t_1, you're at s = s_1. You drive at constant velocity until t = t_2; **how far did you travel? i.e. what is s_2?**

$$s_2 = s_1 + v \times (\text{time travelled})$$

$$\text{Or, } s_2 = s_1 + v\Delta t \quad \text{where} \quad \Delta t = t_2 - t_1$$

What does this look like on a graph of velocity vs. time?

How to get Position from Velocity



$$s_2 = s_1 + v\Delta t \quad \text{where} \quad \Delta t = t_2 - t_1$$

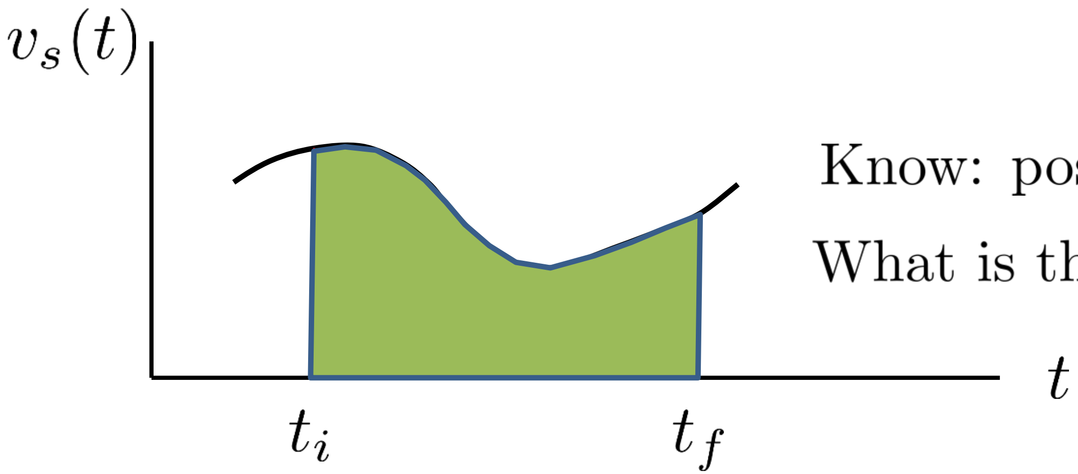
$$\text{Or, } s_2 = s_1 + \text{ (area under the } v(t) \text{ curve from } t_1 \text{ to } t_2)$$

If you have velocity for some time, you ***accumulate*** position.
In calculus, we call this accumulation, **integration**.

$$\text{(area under the } v(t) \text{ curve from } t_1 \text{ to } t_2) = \int_{t_1}^{t_2} v(t)dt$$

How to get Position from Velocity - General

$$(i.e.\ \ v_s \neq constant)$$



Know: position s_i at time t_i

What is the position s_f at time t_f?

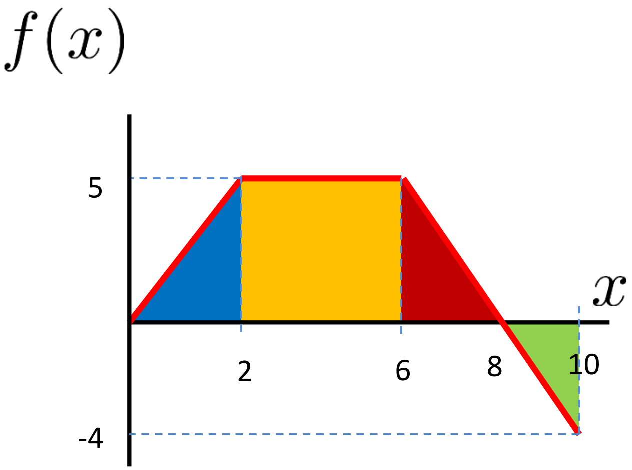
$$\text{The position at time } t_f = s_f = s_i + \int_{t_i}^{t_f} v_s(t)dt$$

$$= s_i + \text{ Area under the } v_s(t) \text{ curve from } t_i \text{ to } t_f$$

What about Integrals?

What kind of integrals will we need to do?

Graphical: e.g. straight lines



$$\int_0^{10} f(x)dx = \text{area}$$

$$= \tfrac{1}{2}(2)(5) + (4)(5) + \tfrac{1}{2}(2)(5) + \tfrac{1}{2}(2)(-4)$$

$$= 26$$

Analytical:

$$f(x) = cx^n$$

(where n and c are constants and $n \neq -1$)

$$\int_{x_1}^{x_2} f(x)dx = \int_{x_1}^{x_2} cx^n dx$$

$$= c\int_{x_1}^{x_2} x^n dx$$

$$= c\left[\frac{x^{n+1}}{n+1}\right]_{x_1}^{x_2}$$

$$= \frac{c}{n+1}\left[x_2^{n+1} - x_1^{n+1}\right]$$

There are many more, but, for now, that's all we'll need!

Whiteboard Problem 2-4

Evaluate the following integrals*, sketch the function, and shade in the integral:

a) $$\int_2^4 5x^2 dx$$

(LC sketch, then calculate)

$$(\text{Answer: } \frac{280}{3} = 93.33)$$

b) $$\int_0^{10} (3 - 6x) dx$$

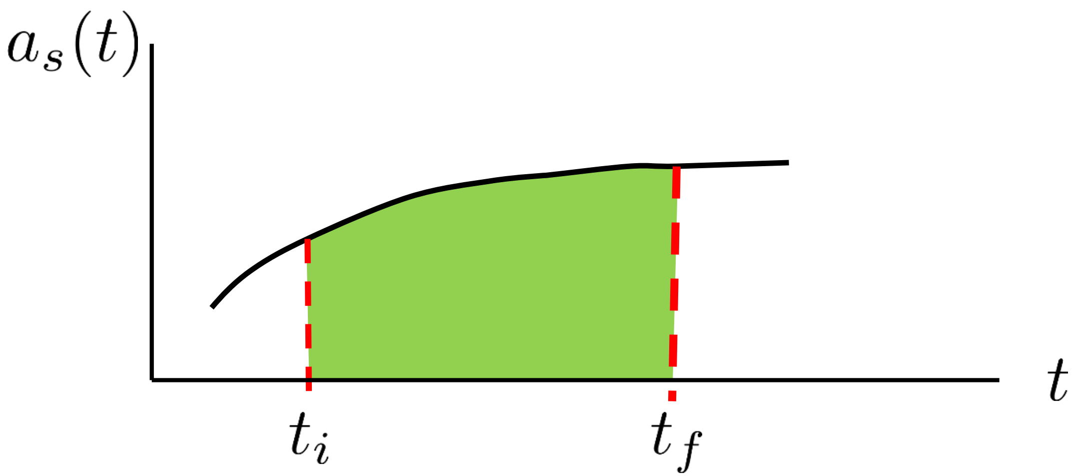
(LC sketch, then calculate)

$$(\text{Answer: } -270)$$

**If you have seen this before in Calculus, show all of the steps on your WB, and help your group members who might be taking Calculus I now.*

How to get Velocity from Acceleration - General

As you might guess, we get the velocity from the acceleration in a similar way:



Know: v_{s_i} at time t_i

What is v_{s_f} at time t_f?

$$\text{The velocity at time } t_f = v_{s_f} = v_{s_i} + \int_{t_i}^{t_f} a_s(t)dt$$

$$= v_{s_i} + \text{Area under the } a_s(t) \text{ curve from } t_i \text{ to } t_f$$

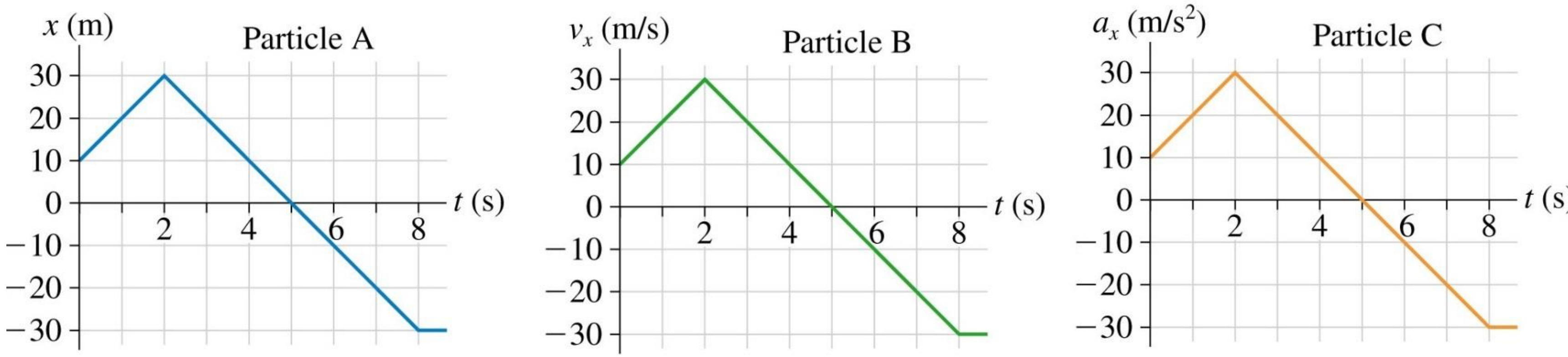
Table Challenge Problem 2 (8/30/23)

Table:

Names:

You are to solve the following problem as a group of your entire table. Work together on your whiteboards and the wall whiteboards. No computers or cell phones are permitted; use only your equation sheet and calculator. When the group has arrived at an answer, write it below and turn this sheet in.
Only your answer will be graded.

Three particles move along the x-axis, each starting with v_{x0} = 10 m/s at t_0 = 0 s. In the figures below, the graph for A is a position vs. time graph; the graph for B is a velocity vs. time graph; the graph for C is an acceleration vs. time graph. **Find each particle's velocity at t = 7.0 s.** (Work with the geometry of the graphs, not any kinematic equations.)



Answer : ____________ **Answer :** ____________ **Answer :** ____________