Motion in One Dimension*

Motion along the s-axis (why s? Here, your author lets **s** stand for either **x** or **y**):



S = object's **position** coordinate along the s-axis (can be +, -, or 0)

- **V**_s = object's **velocity** along the s-axis (can be +, -, or 0)
- **a**_s = object's **acceleration** along the s-axis (can be +, -, or 0)

*Note, in **one dimension**, we can drop the vector notation. Looking ahead, these are the **s-components** of the **vectors** \vec{r} , \vec{v} , and \vec{a}

Motion in One Dimension

In Chapter 1, we introduced motion diagrams to describe motion. Now, we want to represent the motion as <u>mathematical functions</u> of time

$$s(t), v_s(t), \text{ and } a_s(t)$$

e.g. we may have something like this, i.e. graphs of the functions:



Note, these are graphs – not pictures. The object is still moving in one dimension along the s-axis. Our goal in Chapter 2 is to develop the mathematical relations between s(t), $v_s(t)$, and $a_s(t)$.

General 1D Motion: Average Velocity

 $(a_s = a_s(t) \neq \text{constant})$



Whiteboard Problem: 2-1

The figure below is the position vs. time graph of a jogger. What is the jogger's velocity at

a) t = 10s (LC)
b) t = 25s (LC)
c) t = 35 s (LC)



General 1D Motion: Instantaneous Velocity

 $(a_s = a_s(t) \neq \text{constant})$

So far we have:

<u>Average Velocity</u> between $t_1 \& t_2 = v_{s_{avg}} = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$

= <u>slope</u> of the line connecting points 1 & 2



How do we find the velocity at a given instant of time **t**?

We let t₁ and t₂ get closer and closer together until they're infinitesimally close to t.

The line connecting t_1 and t_2 when they are infinitesimally close to t is the <u>tangent line</u>.

General 1D Motion: Instantaneous Velocity



Note: ds and dt are Δs and Δt when the points are infinitesimally close.

What about Derivatives?

Derivatives (from Calculus*)

Slope of a straight line:

y

Slope for any point on the line

Х

$$=\frac{y_2-y_1}{x_2-x_1}=\frac{\Delta y}{\Delta x}$$

Derivative = constant = slope between 1 and 2.

Curve: what is the slope at x?



As the two points used for the slope get closer together

$$\frac{\Delta y}{\Delta x} \to \frac{dy}{dx}$$

 $\frac{dy}{dx}$ is called the **derivative** of the function y. (the derivative is a function too.)

It gives the slope of the tangent line at the point x, or the instantaneous rate of change of the function at the point x.

*If you haven't seen this in Calculus yet – you will very soon. There, you'll do all the theorems, proofs, etc. Here, we just need to understand what a derivative is and how to calculate some very simple ones.

What kinds of derivatives will we need to do?

Graphical:



Horizontal straight line:

$$y(x) = \text{constant}$$

Analytical:

Straight line with slope m:

$$y(x) = mx + b$$

$$\frac{dy}{dx} = m$$

 $\frac{dy}{dx} = 0$

Power law function:

$$y(x) = cx^n$$

(c & n are constants)

$$\frac{dy}{dx} = cnx^{n-1}$$

Sum of two functions:



We'll have other rules as we need them.

Whiteboard Problem 2-2

Determine expressions for the derivatives of the following functions.* (Enter your <u>expressions</u> on LC)

a)
$$f(x) = 10x^3$$

(Answer: $30x^2$)
b) $f(x) = \frac{6}{x} + 5x$
(Answer: $-\frac{6}{x^2} + 5$)

Now, a quick review for what this means for velocity:

*If you have seen this before in Calculus, <u>show all of the steps on your WB</u>, and help your group members who might be taking Calculus I now.

A Quick Review of What We Have for Velocity



Acceleration is the Rate of Change of Velocity

Now that we know a little about slopes and derivatives, the acceleration is obtained from the velocity in a similar manner.



= instantaneous rate of change of v_s(t)

Whiteboard Problem: 2-3

A particle moving along the x-axis has its position described by

x = (2t² - t +1) meters

where t is in seconds.

At t = 2s, what are the particle's

- a) position in m (LC)
- b) velocity in m/s (LC)
- c) acceleration in m/s² (LC)

How to get Position from Velocity

We've seen how to get velocity from position: $v_s(t) = \frac{ds}{dt}$

How do we get position from velocity? This is something you do everyday, *e.g. suppose you're driving in your car at constant velocity:*



At time t_1 , you're at $s = s_1$. You drive at <u>constant velocity</u> until $t = t_2$; **how far did you travel? i.e. what is s_2?**

$$s_2 = s_1 + v \times (\text{time travelled})$$

Or,
$$s_2 = s_1 + v\Delta t$$
 where $\Delta t = t_2 - t_1$

What does this look like on a graph of velocity vs. time?

How to get Position from Velocity



 $s_2 = s_1 + v\Delta t$ where $\Delta t = t_2 - t_1$

Or, $s_2 = s_1 + (\text{area under the } v(t) \text{ curve from } t_1 \text{ to } t_2)$

If you have velocity for some time, you *accumulate* position. In calculus, we call this accumulation, integration.

(area under the
$$v(t)$$
 curve from t_1 to t_2) = $\int_{t_1}^{t_2} v(t) dt$





What kind of integrals will we need to do?



There are many more, but, for now, that's all we'll need!

Whiteboard Problem 2-4

Evaluate the following integrals*, sketch the function, and shade in the integral:

a)
$$\int_{2}^{4} 5x^2 dx$$

(LC sketch, then calculate) (Answer: $\frac{280}{3} = 93.33$)

b)
$$\int_{0}^{10} (3-6x) dx$$
 (LC sketch, then calculate)
(Answer: -270)

*If you have seen this before in Calculus, <u>show all of the steps on your WB</u>, and help your group members who might be taking Calculus I now.

How to get Velocity from Acceleration - General

As you might guess, we get the velocity from the acceleration in a similar way:



Whiteboard Problem 2-5

Three particles (A, B, & C) move along the x-axis, each starting with $v_{x0} = 10 \text{ m/s}$ at $t_0 = 0 \text{ s}$. In the figures below, graphs are shown of x(t), v(t), and a(t).

Find each particle's velocity at t = 7.0 s <u>working only with the</u> <u>geometry of the graphs</u>, not any kinematic equations.

- a) Find Particle A's velocity at t = 7.0s (LC)
- **b)** Find Particle B's velocity at t = 7.0s (LC) (easiest one)
- c) Find Particle C's velocity at t = 7.0s (LC) (hardest one)

