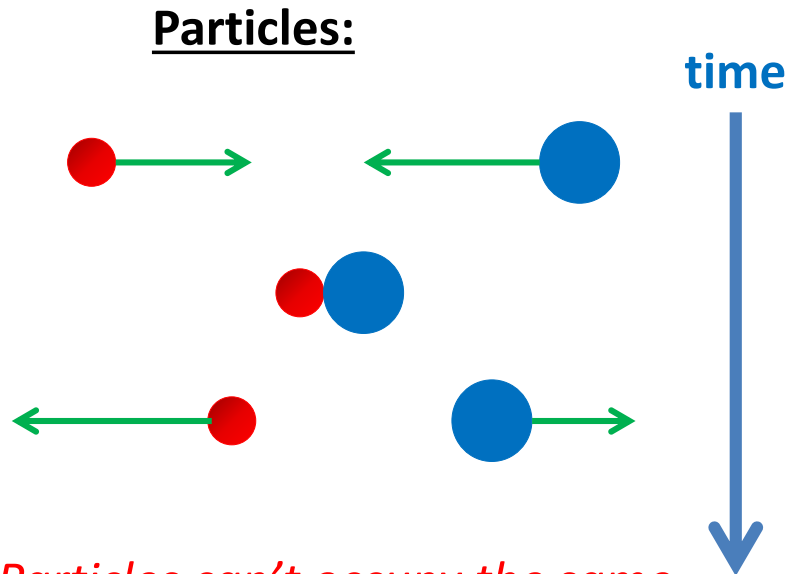
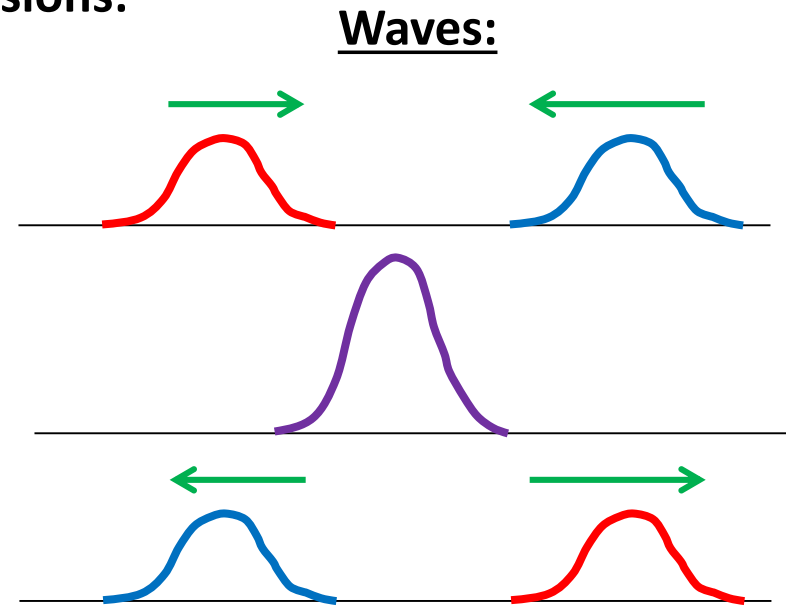


17: The Superposition of Waves

Perhaps the most important difference between particle behavior and wave behavior is what happens during collisions:



Particles can't occupy the same space; so they bounce off each other



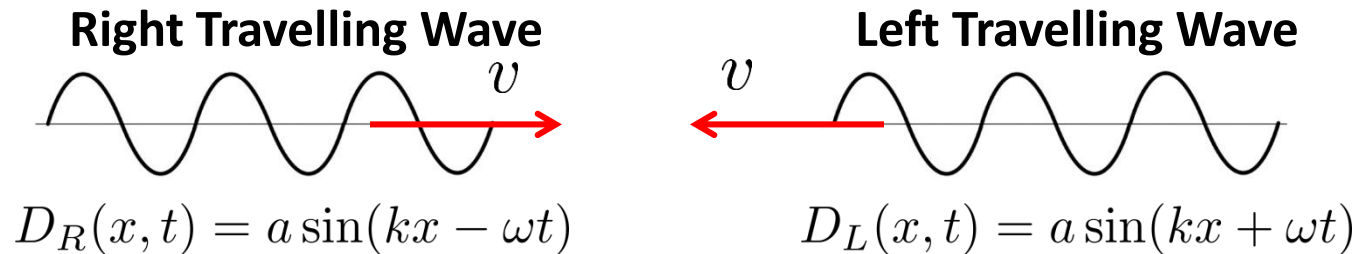
Waves overlap, add together, and pass right through each other.

Principle of superposition When two or more waves are *simultaneously* present at a single point in space, the displacement of the medium at that point is the sum of the displacements due to each individual wave.

Standing Waves

We can form a **Standing Wave** by superposing (i.e. adding together) two sinusoidal travelling waves of the same amplitude, wavelength, and frequency, but travelling in opposite directions.

The name **Standing Wave** comes from the fact that the resultant wave has an amplitude that oscillates in place.



The Resultant Wave: $D(x, t) = D_R(x, t) + D_L(x, t)$
↓ (some math, details in your text)

$$D(x, t) = \underbrace{2a \sin kx}_{A(x), \text{ amplitude function}} \cos \omega t$$

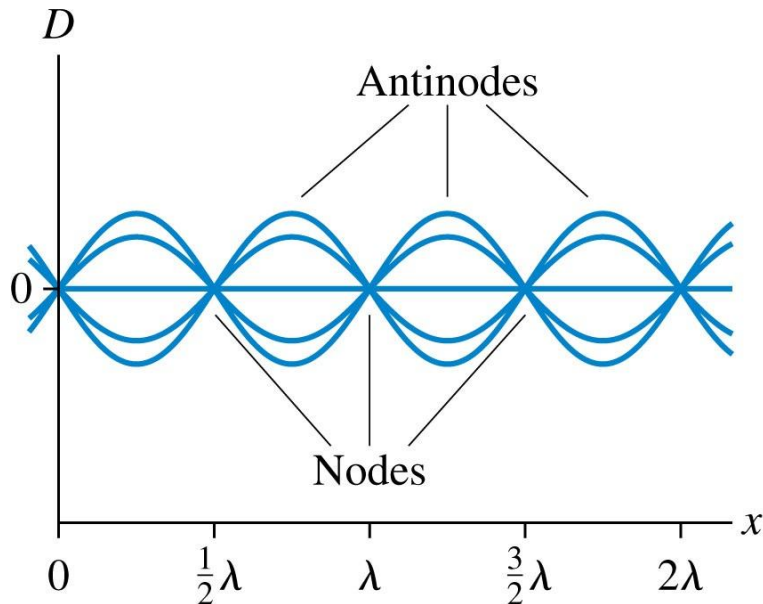
What does
this look like?

Standing Wave Terminology

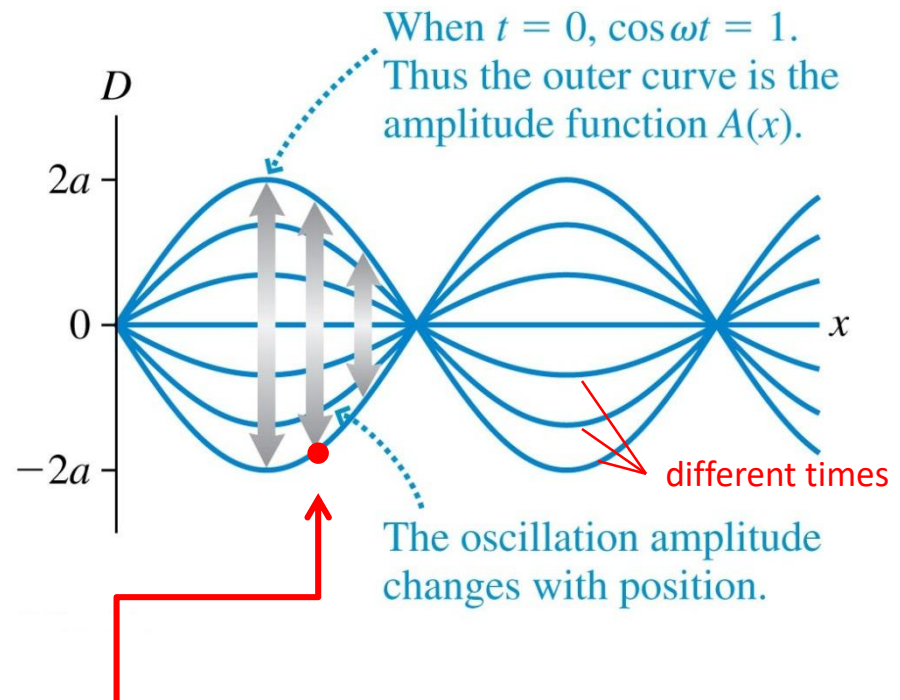
$$D(x, t) = 2a \sin kx \cos \omega t = 2a \sin \frac{2\pi x}{\lambda} \cos \omega t = A(x) \cos \omega t$$

When $\frac{x}{\lambda} = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \dots \Rightarrow$ No displacement, a **Node**

When $\frac{x}{\lambda} = \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \dots \Rightarrow$ Displacement is maximum, an **Antinode**



The nodes and antinodes are spaced $\lambda/2$ apart.



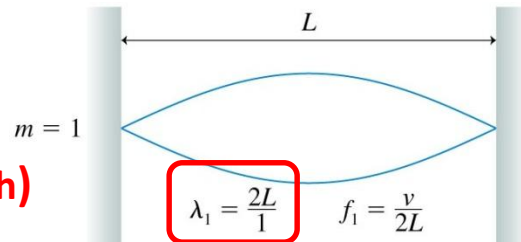
A point here has SHM with Amplitude $A(x)$

Standing Waves on a String

For a string clamped at both ends, what are the wavelengths and frequencies of the allowed standing waves? *The two waves that combine are the original wave and its reflected wave.*

Standing Wave Condition: Both ends must be a node. So an integer number of half wavelengths can fit on the string.

**“Fundamental”
(longest wavelength)**



So, following the pattern:
The Allowed Wavelengths are:

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, 4, \dots$$

And the allowed frequencies
(v = wavespeed) are:

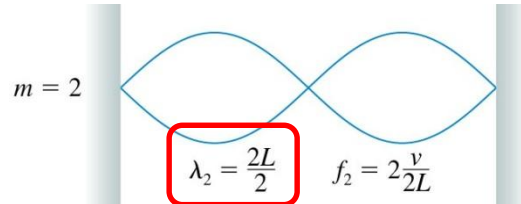
$$f_m = \frac{v}{\lambda_m} = m \frac{v}{2L} = m f_1$$

where: $f_1 = \frac{v}{2L}$

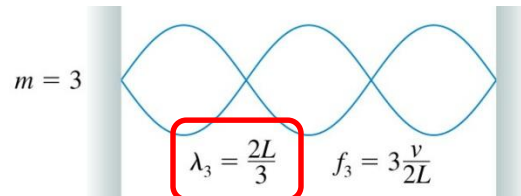
Note the important fact:

The m^{th} harmonic
has m antinodes.

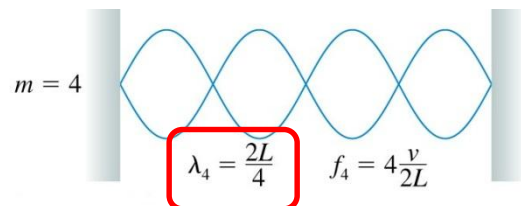
**“Second
Harmonic”**



**“Third
Harmonic”**



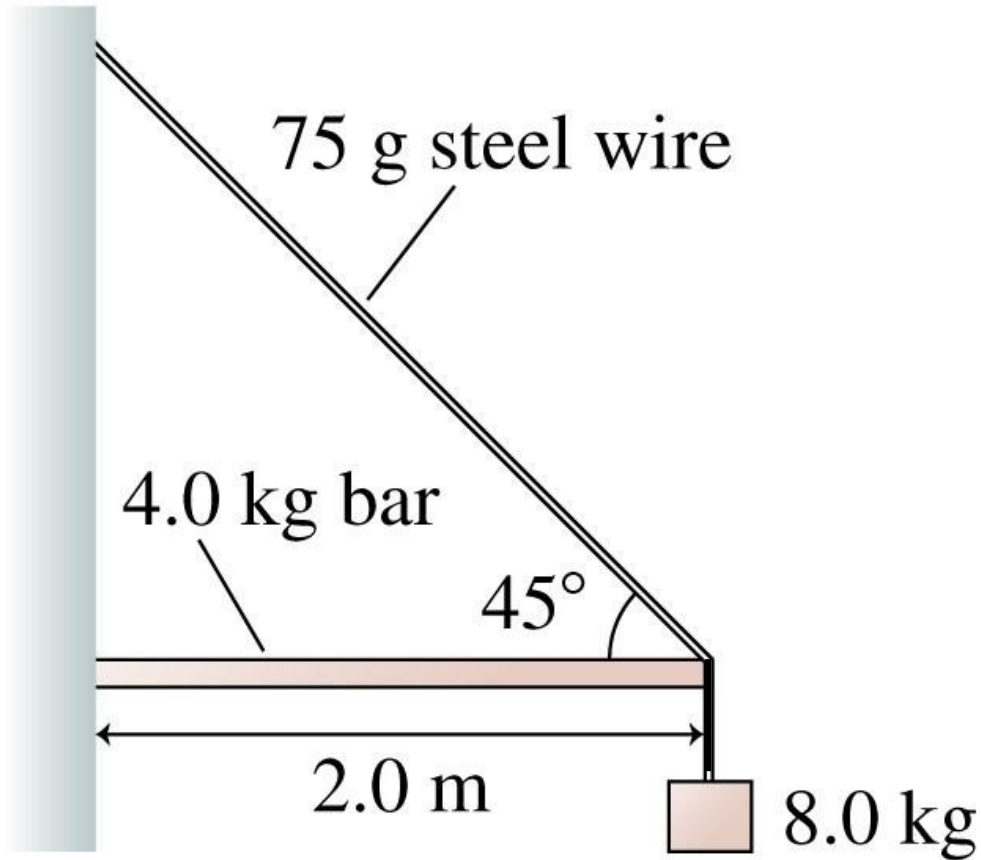
**“Fourth
Harmonic”**



Whiteboard Problem 17-1

(A good review problem – a two point LC shot!)

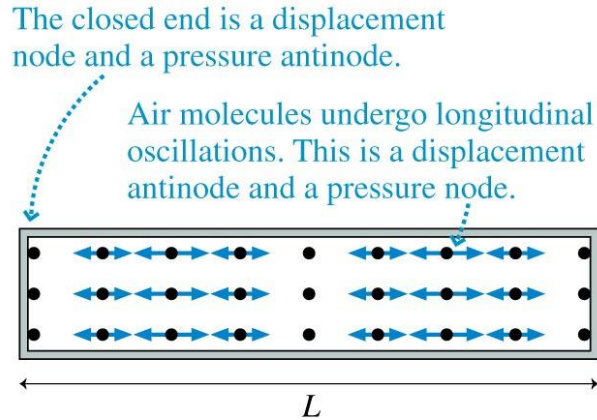
What is the fundamental frequency for standing waves on the steel wire in the figure if the system is in equilibrium? (LC)



[Here's a neat video](#) of standing waves produced on a 2D oscillating plate and a good explanation of standing waves in general. 5

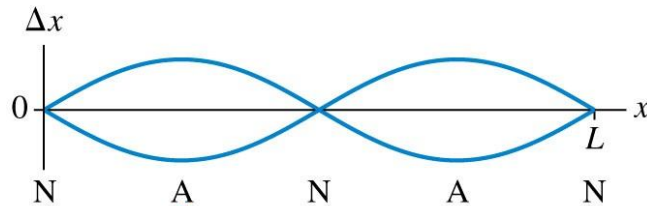
Standing Sound Waves

One dimensional standing waves can be set up in a column of gas. Here, we can picture the wave as a pressure wave or a longitudinal displacement wave:

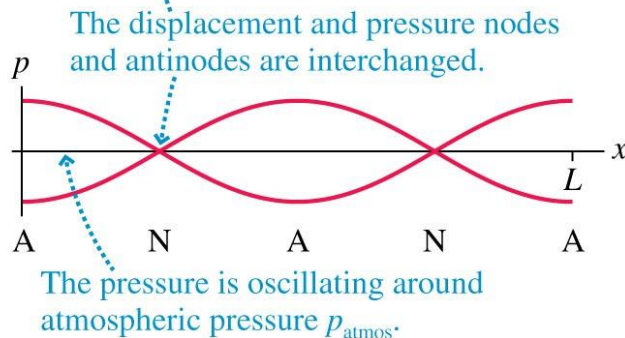


Tube closed at both ends
with a second harmonic wave

Actual oscillations of the molecules



The displacement amplitude of the molecules (looks like a wave on a string)

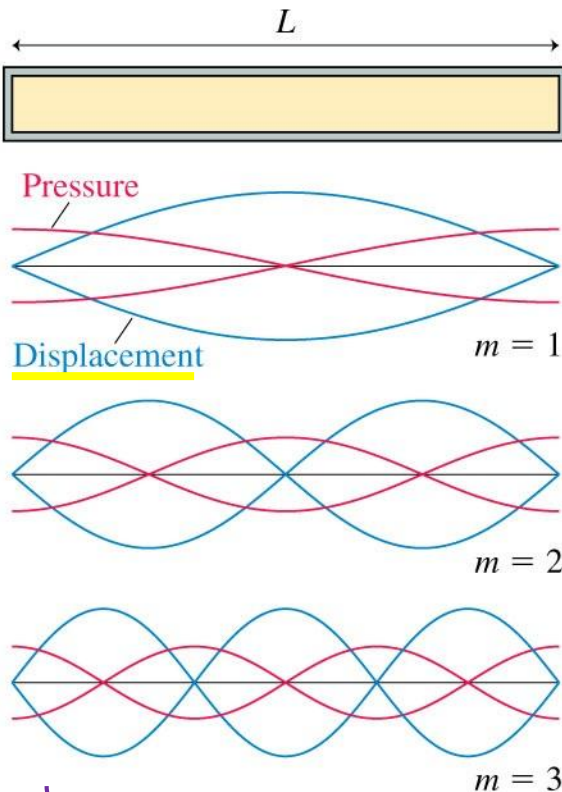


The pressure amplitude

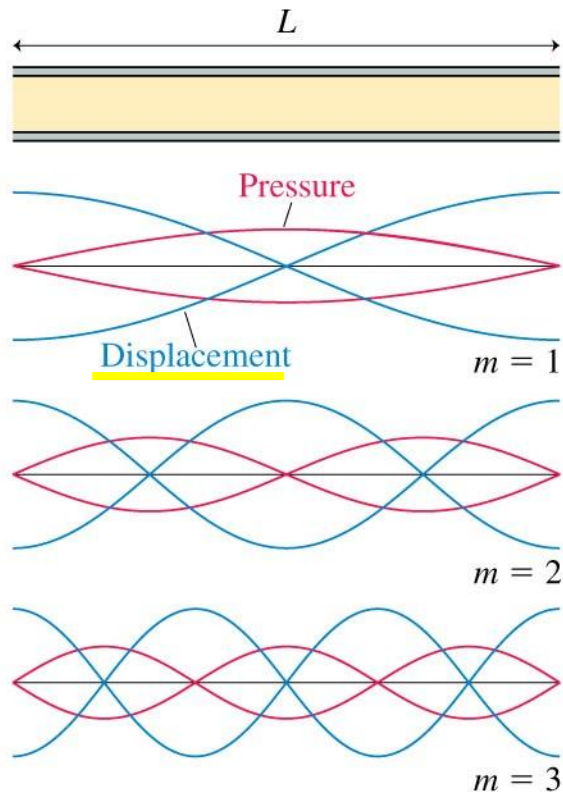
Since the displacement amplitude behaves just like it does for a string, we work with it, but remember the pressure nodes and antinodes are different.

Allowed Standing Waves in Tubes

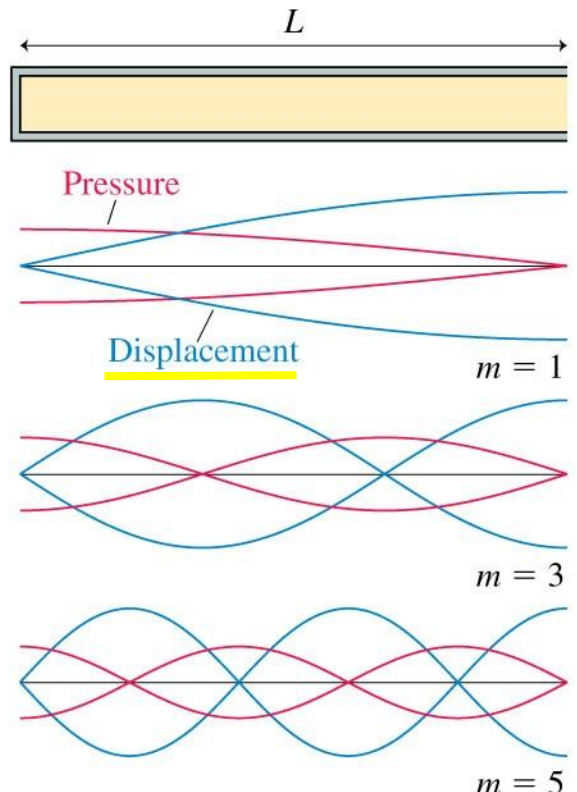
(a) Closed-closed



(b) Open-open



(c) Open-closed



Look at Displacement Amplitude; for both of these, :
(fit an integer number of half wavelengths in the tube)

$$\lambda_m = \frac{2L}{m} \quad m = 1, 2, 3, 4, \dots$$

$$f_m = \frac{v}{\lambda_m} = m \frac{v}{2L} = m f_1$$

(exactly like a string) $v = \text{sound speed}$

For this one:

(fit an odd integer number of quarter wavelengths in the tube)

$$\lambda_m = \frac{4L}{m} \quad m = 1, 3, 5, 7, \dots$$

$$f_m = \frac{v}{\lambda_m} = m \frac{v}{4L} = m f_1$$

Whiteboard Problem 17-2

What are the three longest wavelengths for standing sound waves in a 121 cm long tube that is

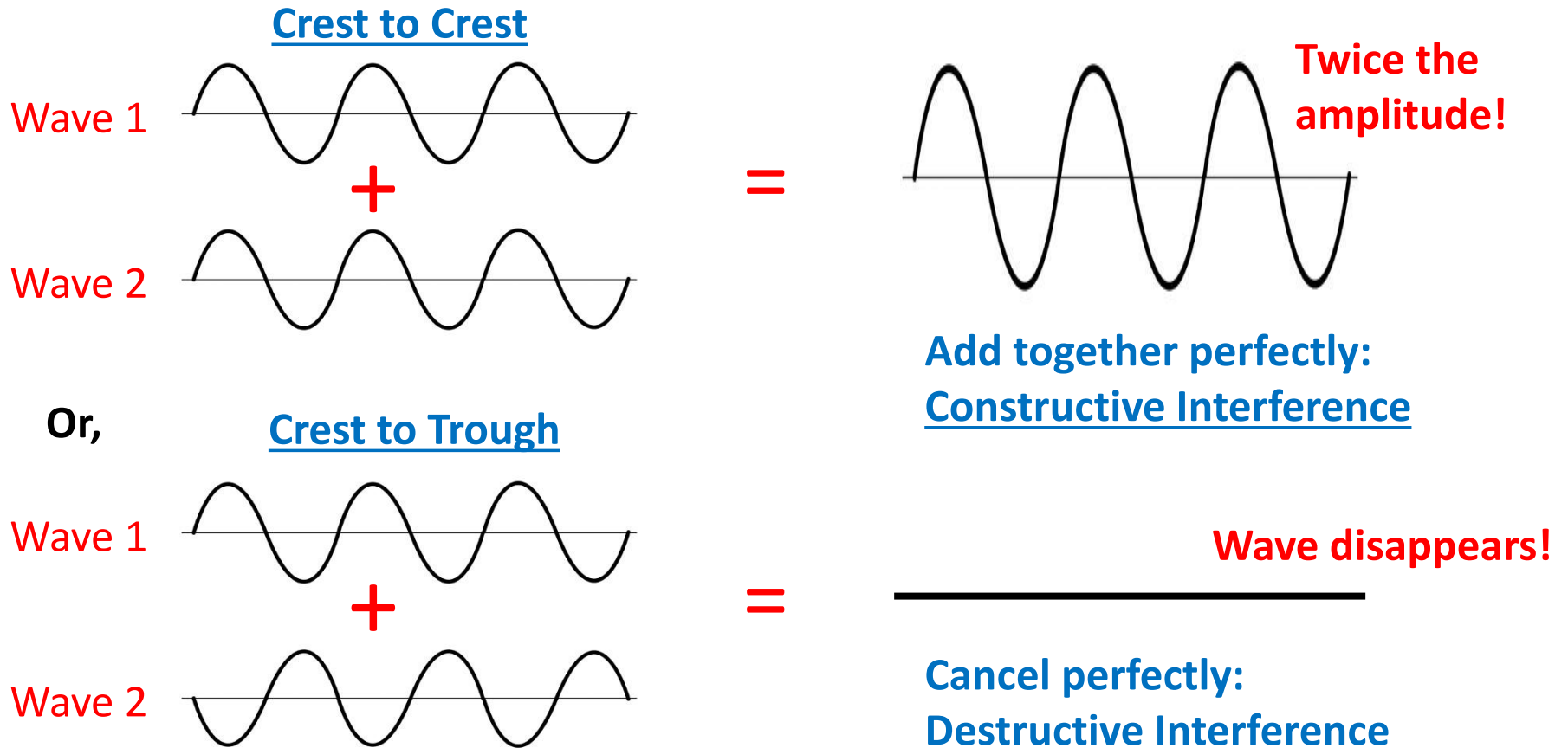
- a) open at both ends? (LC: enter longest wavelength in cm)
- b) open at one end and closed at the other?
(LC: enter longest wavelength in cm)

Of course, standing waves on strings and in tubes of air are the basis of many musical instruments. Your author has a good discussion of this on page 482-483. If you're interested, you may want to consider taking PHY131, The Physics of Music.

Here is a really splendid demo of standing sound waves in a tube (a [Reuben's Tube](#)).

Wave Interference in 1D

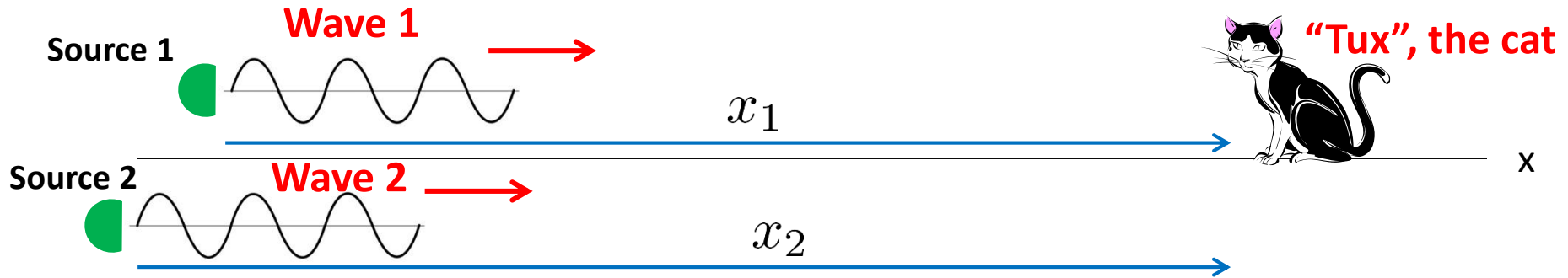
We've seen that when two waves add together (*superpose*), there is a possibility of:



In general, when two waves add together, you don't get anything interesting, **but these two conditions are worth a closer look.**

Wave Interference in 1D

Consider two waves with the **same amplitude, frequency, and wavelength** that both travel in the +x direction.



At the location of Tux, the cat:

$$\left. \begin{aligned} \text{wave 1: } D_1(x_1, t) &= a \sin(kx_1 - \omega t + \phi_{0_1}) = a \sin \phi_1 \\ \text{wave 2: } D_2(x_2, t) &= a \sin(kx_2 - \omega t + \phi_{0_2}) = a \sin \phi_2 \end{aligned} \right\} \phi \text{ is the total phase}$$

Tux observes the combined wave, and the way the waves combine at his location is totally determined by the **Phase Difference**.

$$\Delta\phi = \phi_2 - \phi_1$$

Perfect Constructive Interference for: $\Delta\phi = 0, 2\pi, 4\pi, \dots = m \cdot (2\pi)$

Perfect Destructive Interference for: $\Delta\phi = \pi, 3\pi, 5\pi, \dots = (m + \frac{1}{2}) \cdot (2\pi)$

where: $m = 0, 1, 2, 3, \dots$

Wave Interference in 1D

Now, from the expressions for the two waves detected by Tux, the cat:

$$\begin{aligned}\Delta\phi &= (kx_2 - \omega t + \phi_{0_2}) - (kx_1 - \omega t + \phi_{0_1}) \\ &= k(x_2 - x_1) + (\phi_{0_2} - \phi_{0_1}) \\ &= \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0\end{aligned}$$

So, the two waves can have a phase difference for two reasons: a path difference; and an initial (inherent) phase difference.

So, we have the two Interference Conditions:

Maximum Constructive Interference: $\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0 = m \cdot 2\pi \text{ rad}$

Perfect Destructive Interference: $\Delta\phi = \frac{2\pi}{\lambda}\Delta x + \Delta\phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}$

where for both: $m = 0, 1, 2, 3, \dots$

Note: for two sources **initially in phase**, these conditions say that there's **constructive interference when the path difference is an integral number of wavelengths**, and **destructive interference when the path difference is a half integral number of wavelengths**.

An application: [have you seen this commercial from a few years ago? . . . Here's how they work.](#)

Whiteboard Problem 17-3

Two loudspeakers in a 20°C room emit 686 Hz sound waves along the x axis. An observer is located at $x = x_0$.

- a) If the speakers are “*in phase*”, what is the smallest distance between the speakers for which the interference of the sound waves is perfectly destructive at the location of the observer? (LC)
- b) If the speakers are “*out of phase*”, what is the smallest distance between the speakers for which the interference of the sound waves is maximally constructive at the location of the observer? (LC)

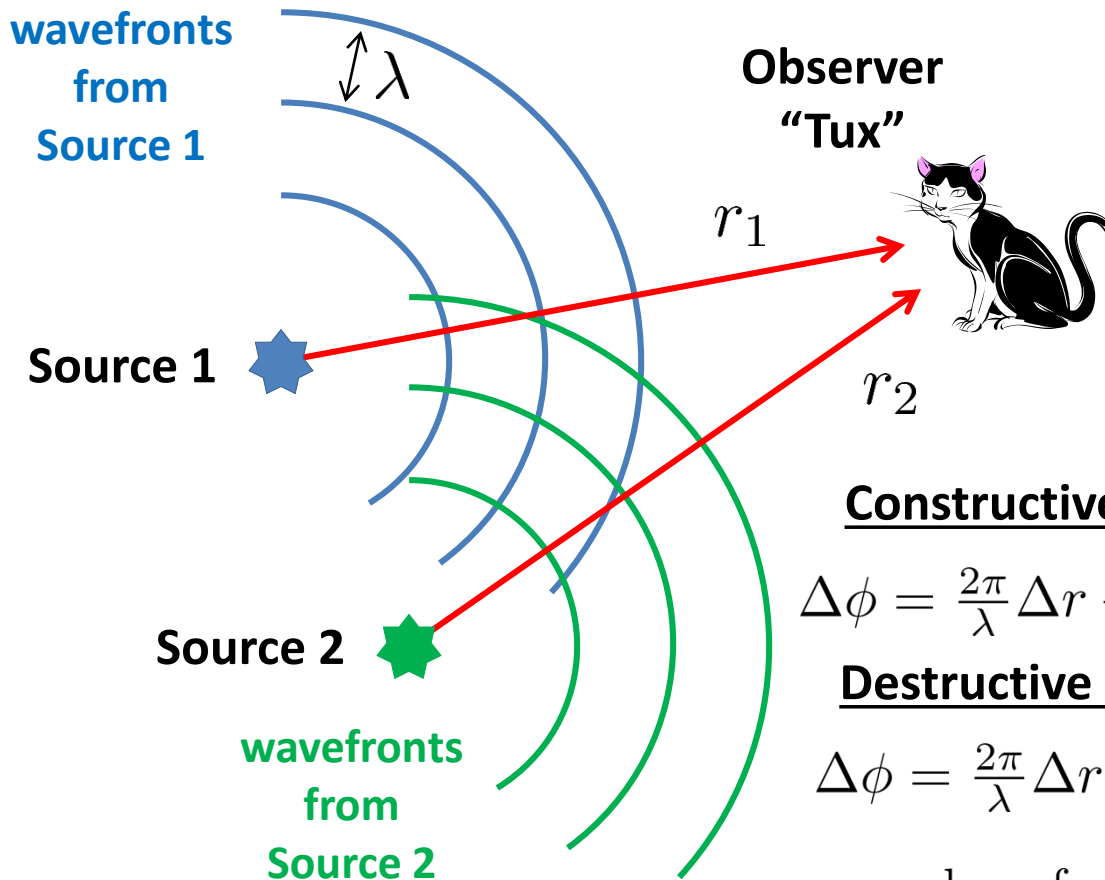
Note: “In Phase” $\Rightarrow \Delta\phi_0 = 0$

“Out of Phase” $\Rightarrow \Delta\phi_0 = \pi$

Try doing this problem by two ways: first use the formulae on the previous slide, then draw it with the waves, and think about how far you should move one of the sources to bring the two waves out of phase (part a) or into phase (part b).

Interference in 2D and 3D

As we did for 1D, consider two waves of the same amplitude and frequency in 2D:



Again, Tux observes the combined wave, and the way the waves combine depends on their phase. The phase difference depends on the path difference and the initial phase.

Constructive Interference:

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta r + \Delta\phi_0 = m \cdot 2\pi \text{ rad}$$

Destructive Interference:

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta r + \Delta\phi_0 = \left(m + \frac{1}{2}\right) \cdot 2\pi \text{ rad}$$

where for both: $m = 0, 1, 2, 3, \dots$

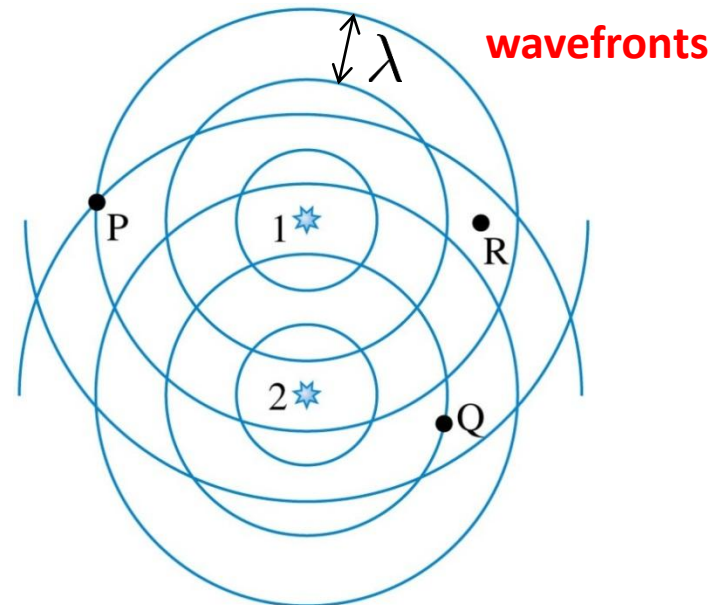
Note: these are the same conditions that we had for 1D, but x is replaced with r . The big difference here is in the geometry.

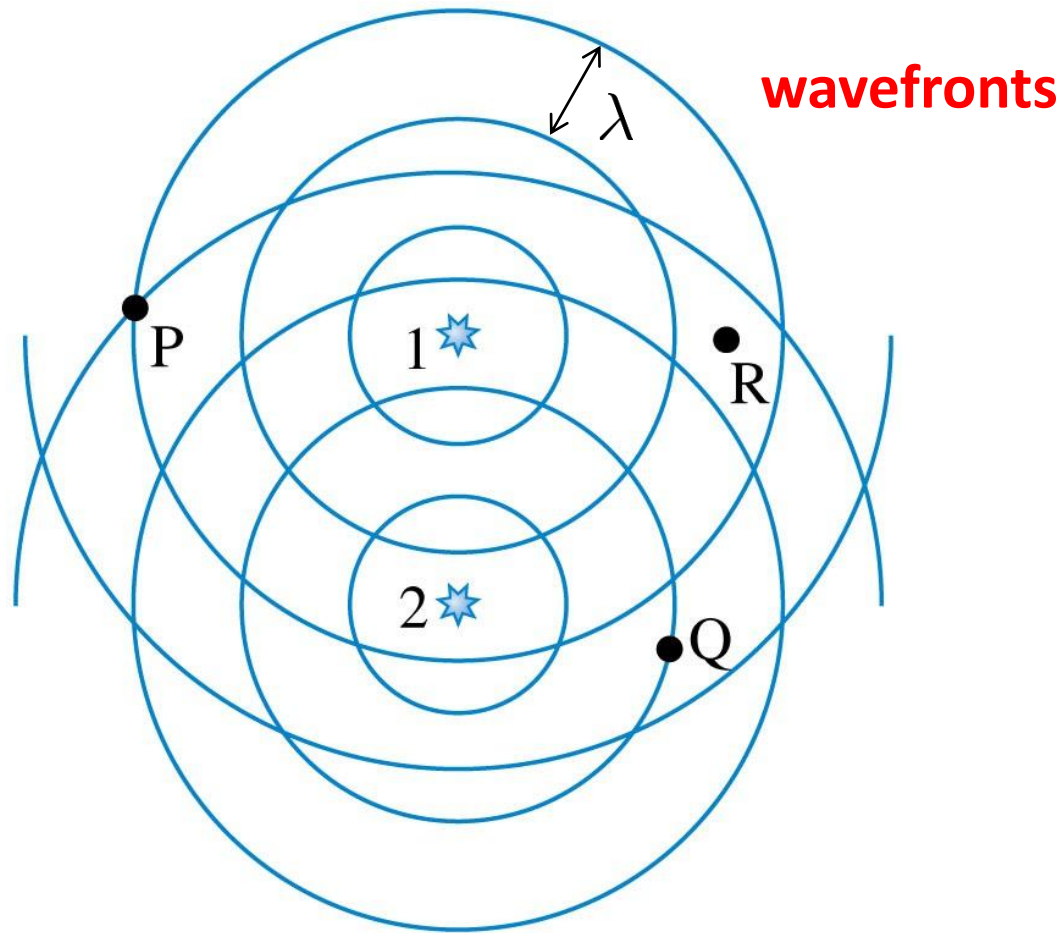
Whiteboard Problem: 17-4

The figure below shows the circular wavefronts emitted by two wave sources labelled 1 and 2 that have the same wavelength.

- Are the sources in phase or out of phase? (LC)
- Use the supplied figure and table. Fill in the table for points P, Q, and R, giving the distances as multiples of the wavelength and indicating, with a C or a D, whether the interference at that point is constructive or destructive.

Use the supplied copy of the figure to determine the distances, and fill in the table with the figure.



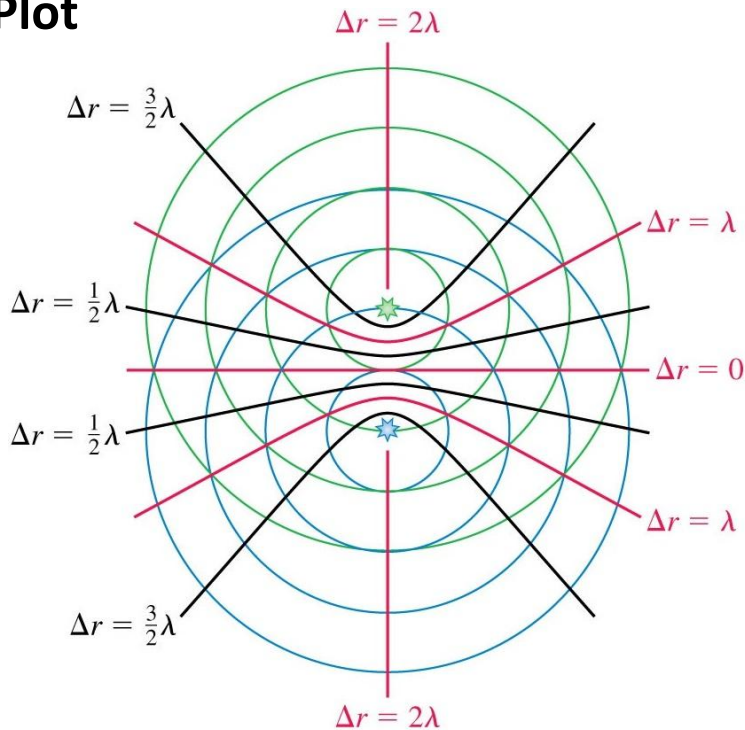


	r_1	r_2	Delta r	C or D
Point P				
Point Q				
Point R				

2D Interference Patterns

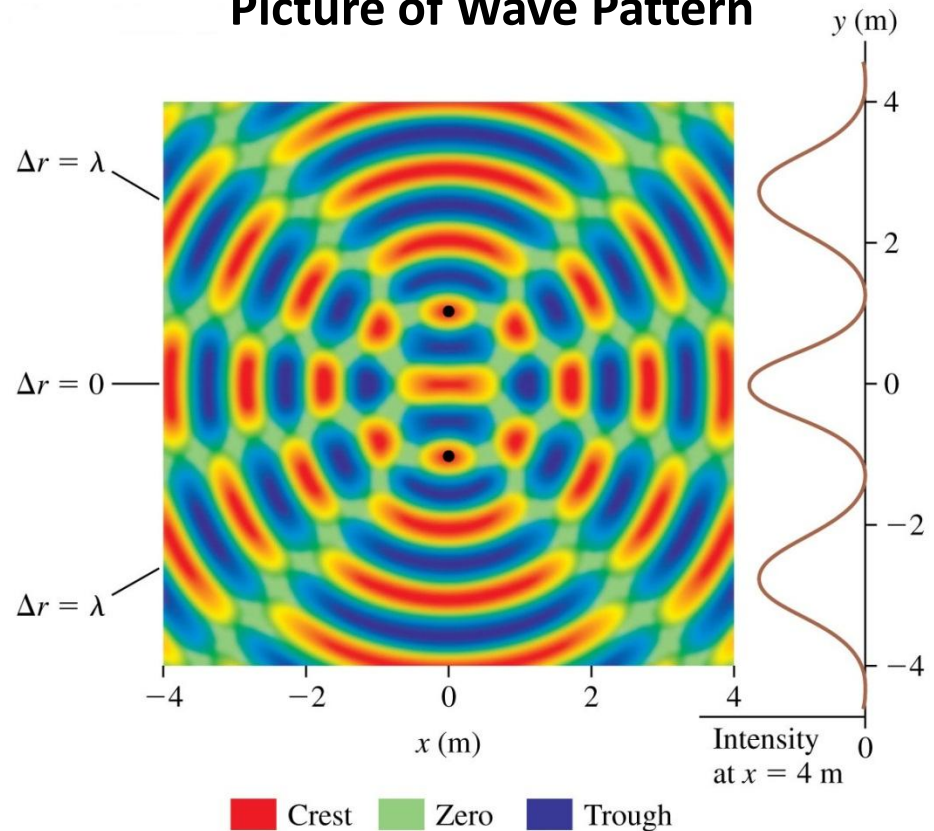
If you extend WB Prob 17-4 over the entire 2D plane, you get patterns of interference. Your Text has some nice figures for **Two In-Phase Sources**:

Plot



- nodal lines, destructive interference, no oscillation. Intensity is zero
- antinodal lines, constructive interference, oscillations with maximum amplitude. Intensity is maximum

Picture of Wave Pattern



Remember, these are waves, so they're moving!

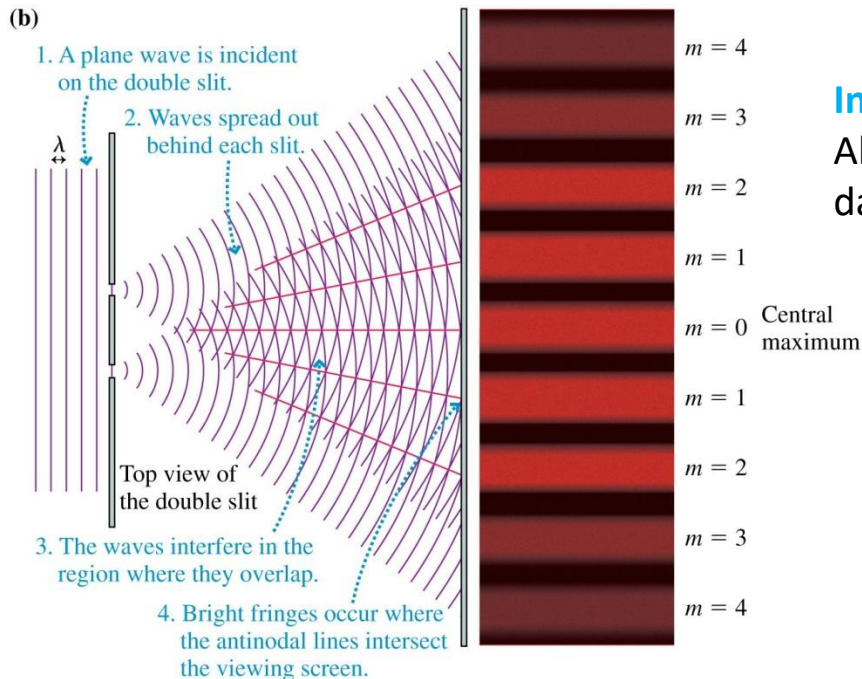
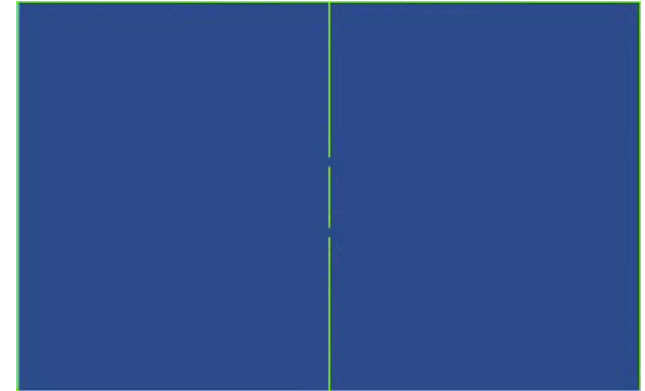
We can see this on a PhET simulation. PhET W Int

Two Slit Interference, an Example of 2D Interference with Light

When a wave of single wavelength encounters two slits, each slit becomes a source of waves that are in phase and radiate outward, where they can interfere.

This is just the 2D interference pattern that we studied in previously.

A two slit pattern generated by light:



Interference Fringes

Alternating bright and dark bands on a screen.

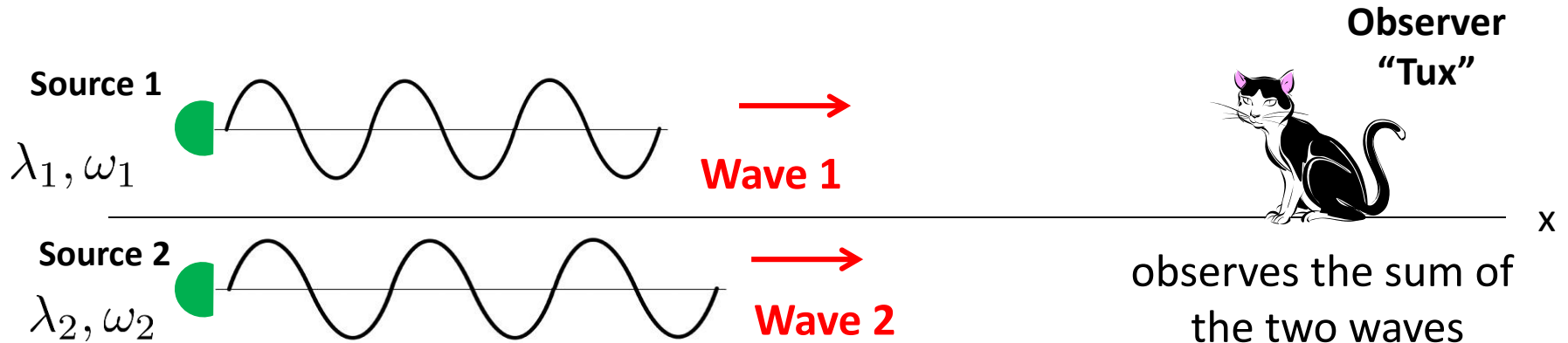
We can see this on the PhET simulation.

You'll play with this simulation in HW, and we'll see the importance of two-slit interference in Quantum Mechanics.

PhET W Int 

Beats

Back to 1D waves. What do we get if we add two waves together that are travelling in the same direction, but have slightly different frequencies?



For the two waves in phase:

$$D_1(x, t) = a \sin(k_1 x - \omega_1 t)$$
$$D_2(x, t) = a \sin(k_2 x - \omega_2 t)$$

At a fixed x , Tux observes the sum of the two waves (*after some math – details in text*)

$$D = D_1 + D_2 = 2a \cos(\omega_{\text{mod}} t) \sin(\omega_{\text{ave}} t)$$

where: $\omega_{\text{mod}} = \frac{1}{2}(\omega_1 - \omega_2)$ **modulation frequency**

$\omega_{\text{ave}} = \frac{1}{2}(\omega_1 + \omega_2)$ **average frequency**

What does this wave look like?

A Beat Wave

For two waves of nearly the same frequency:

$$D = 2a \cos(\omega_{\text{mod}} t) \sin(\omega_{\text{ave}} t)$$

$$\omega_1 \approx \omega_2$$

$$\omega_{\text{mod}} = \frac{1}{2}(\omega_1 - \omega_2)$$

$$\omega_{\text{ave}} = \frac{1}{2}(\omega_1 + \omega_2)$$

Beat Frequency:

$$f_{\text{beat}} = 2f_{\text{mod}} = f_1 - f_2$$

Note, this is a history plot at some position x .

It is actually a wave, so it's moving.

Show on PhET

PhET Fourier 

[Sound Beats Demo:](#)

