16: Wave Motion

Sometimes things move by **travelling oscillations – Waves**.

Consider waves on a pond

Chapter 16 deals with the general properties of waves. We will be considering several different types of waves; however, first we'll concentrate on those characteristics that are common to all waves, then look at sound and light.

First: What is a Wave?

General Definition of a Wave:

"A wave is an organized disturbance that travels at a well defined speed"

Note: the disturbance propagates, the medium (if there is one) has no bulk motion. *(The medium can oscillate, but it doesn't move with the wave.)*

Also, waves are very different than particles which are localized in space (i.e. they exist at a point). Waves are spread out in space, but they do share a feature with particles:

Waves carry energy and momentum.

Types of Waves

We will consider three general types of waves:

1.) Mechanical Waves

- Require a medium to propagate
- The wave speed is determined by the elastic properties and inertia of the medium
- Oscillations can be transverse or longitudinal to the wave direction
- Video of transverse and longitudinal waves on a slinky
- Examples: wave on a string, sound waves, water waves (me playing on a wave last summer), stadium wave(?).

2.) Electromagnetic Waves

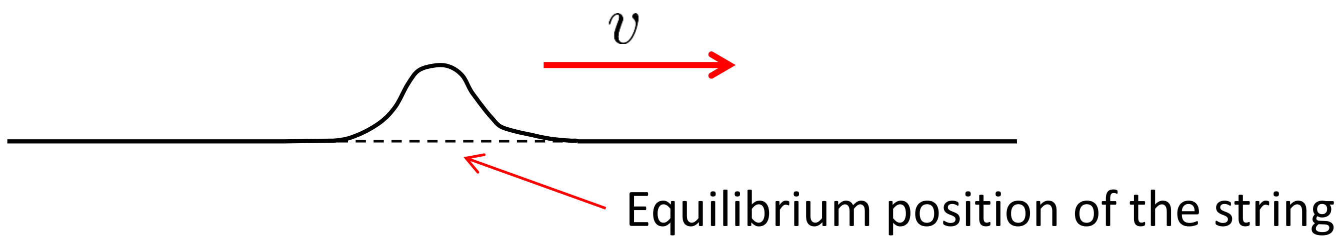
- Requires no medium to propagate (. . . *interesting*)
- Disturbances in the electromagnetic fields that travel in vacuum at the speed of light, $c = 3 \times 10^8$ m/s.
- e.g. visible light, radio waves, x-rays, etc.

3.) Matter Waves

- At the level of fundamental particles, like electrons, particles have wave properties; This is Quantum Mechanics; What's doing the waving?

Wave Speed for a String

Consider a transverse wave pulse on a string:



PhET W on S

As derived in your text, **the speed of the wave is:**

$$\text{Wave Speed, } v = \sqrt{\frac{T_s}{\mu}}$$

Where:

T_s = Tension in the string

μ = linear mass density of the string (mass/length)

PhET W on S

Whiteboard Problem 16-1

The wave speed on a string is 150 m/s when the tension is 75N.

What tension will give a speed of 180 m/s? (LC)

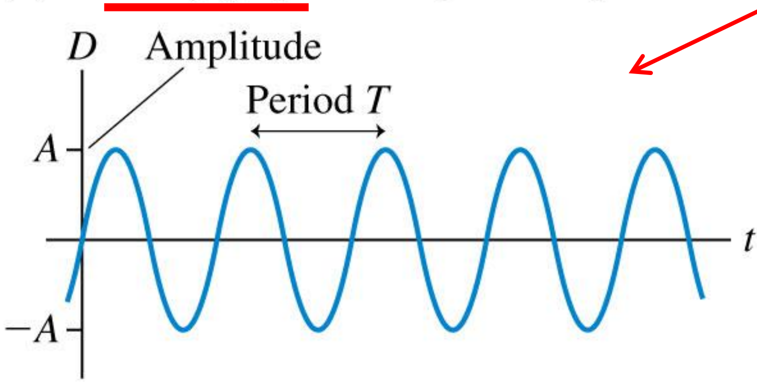
Travelling Sinusoidal Wave

A sinusoidal (or harmonic) disturbance creates a sinusoidal travelling wave.

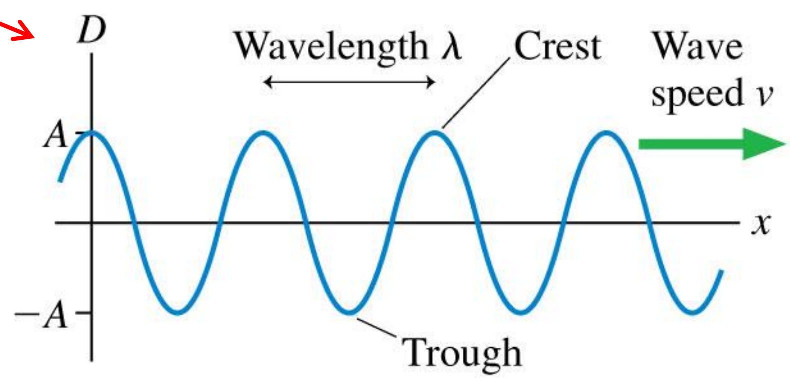
PhET W on S

At a given time, this wave is a sine wave in space, and **at a given point in space**, a point has harmonic motion in time.

(a) A history graph at one point in space



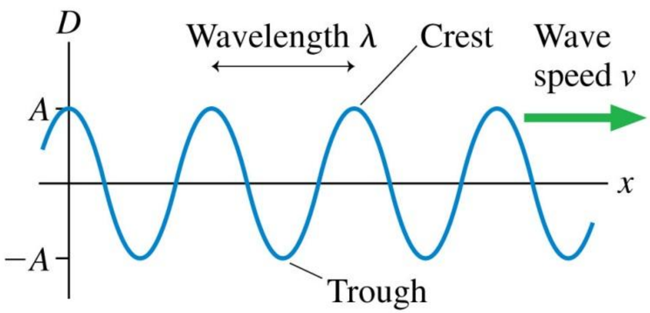
(b) A snapshot graph at one instant of time



Where D(x,t) is the general disturbance from the equilibrium state.
Note: it is a function of two variables.

The Equation of Travelling Sinusoidal Wave

(b) A snapshot graph at one instant of time



$A =$ Amplitude (displacement from undisturbed state)

$v =$ wave speed

$\lambda =$ Wavelength (distance for disturbance to repeat)

$T =$ Period (time for disturbance to repeat)

$f =$ Frequency $= \frac{1}{T}$

"Fundamental Relation for Waves"

$$v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$$

General Equation of a sinusoidal travelling wave:

$$D(x,t) = A\sin(kx - \omega t + \phi_0)$$ **Travelling in +x direction**

$$D(x,t) = A\sin(kx + \omega t + \phi_0)$$ **Travelling in -x direction**

$k =$ Wave Number $= \frac{2\pi}{\lambda}$ [Units $= m^{-1}$]

$\omega =$ Angular Freqency $= 2\pi f$

$\phi_0 =$ phase constant

Watch your k's! This k is not the spring constant.

Whiteboard Problem 16-2

The displacement of a wave traveling in the positive x direction is

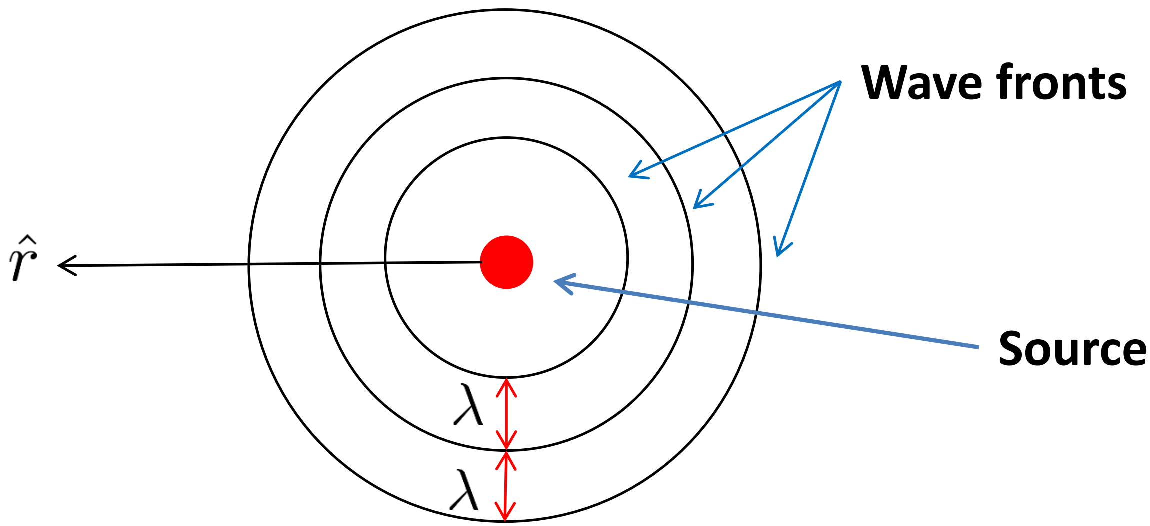
$$D(x,t) = (3.5\ \text{cm})\ \sin(2.7x - 124\ t)$$

where x is in meters and t is in seconds.

What are the:

a) **frequency and wavelength? (LC)**
b) **speed of the wave? (LC)**
c) **displacement D at x = 5.2m and t = 3.6s? (LC)**

Waves in 2 and 3 Dimensions



waves on a pond again.

In 2D or 3D, the amplitude of the wave will decrease since the energy is spread out over a larger circle (in 2D) or a sphere (in 3D). So a sinusoidal wave looks like:

$$D(r,t) = A(r)\sin(kr - \omega t + \phi_0)$$

Note: here, the amplitude can change, and this is for an outgoing wave.

Wave Phase

For any sinusoidal wave (e.g. 1D):

$$D(x,t) = A\sin(\underbrace{kx - \omega t + \phi_0}_{\text{wave ``phase'', } \phi})$$

Don't confuse the wave phase, ϕ, with the phase constant, ϕ_0. ϕ_0 gives the phase at $x = t = 0$.

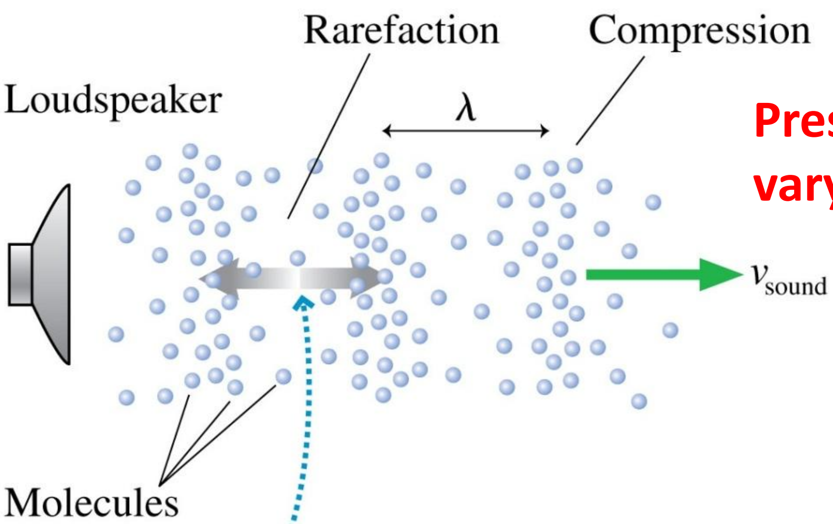
The wave phase determines where you are on the wave, i.e. a peak, zero, trough, or somewhere in between.

In chapter 17, we'll look at combining waves, and the difference in phase will be very important.

Sound Waves

Sound waves are longitudinal compression waves that propagate through a medium – gas, liquid, or solid.

Sound Waves in Air:



Individual molecules oscillate back and forth with displacement *D*. As they do so, the compressions propagate forward at speed v_{sound}. Because compressions are regions of higher pressure, a sound wave can be thought of as a pressure wave.

Pressure and density vary sinusoidally.

PhET

Wave Int.
with sound
& one source

The **speed of sound** in a gas is slightly temperature dependent, for air we will use:

$$v_s \ (\text{at } T = 20°C) = 343 \ m/s$$

Human frequency audible range:

$$\sim 20 \ Hz \ \leq f \leq \ \sim 20 \ kHz$$

Whiteboard Problem 16-3: Breathing from a Helium Balloon

Have you ever breathed Helium from a balloon?
Why does this happen? **Assuming that the typical human voice is at a frequency of 500 Hz, use the sound speeds in Table 20.1 to find the frequency of the human voice after breathing Helium.(LC)**

TABLE 20.1 The speed of sound

Medium	Speed (m/s)
Air (0°C)	331
Air (20°C)	343
Helium (0°C)	970
Ethyl alcohol	1170
Water	1480
Granite	6000
Aluminum	6420

Every time I teach about waves, I have this classic song from the 70s running through my head:

You can probably tell that the name of the song is "Wavelength", but who is the artist? (LC)

This is actually an approximation. Helium in your vocal tract causes it to resonate at higher frequencies so those frequencies dominate what is emitted.

Whiteboard Problem: 16-4

Have you noticed that it is fairly easy to localize the source of a sound, i.e. be able to tell where it comes from. **(Let's try it.)**

a) How do we do this?

The slight difference in arrival times of the sound to your ears. This is also why dogs turn their heads to different angles when they hear a strange sound.

b) Assume that your ears are spaced approximately 20 cm apart. Consider a sound source 5.0 m from the center of your head along a line 45° to your right. What is the difference in arrival times of the sound to your two ears. (LC)

Have you ever had to determine the direction of the source of a sound underwater? It's impossible; the sound seems to come from all directions.

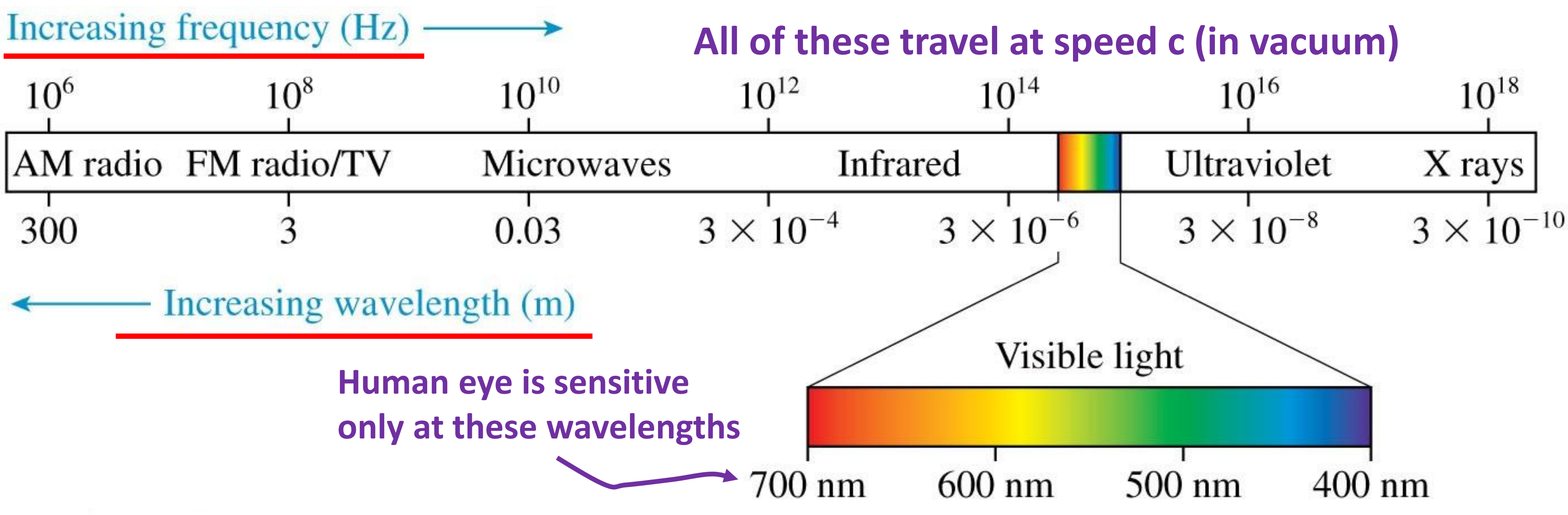
Repeat the calculation for sound in water (if you did your math symbolically, you can just plug in the appropriate numbers.) In water the speed of sound is 1480 m/s.

Light, an Electromagnetic Wave

Light is a disturbance in the electromagnetic fields* created by oscillating electric charges. All electromagnetic waves propagate at:

$$\text{Speed of light in vacuum, } c = 3 \times 10^8 \; m/s$$

The Electromagnetic Spectrum:

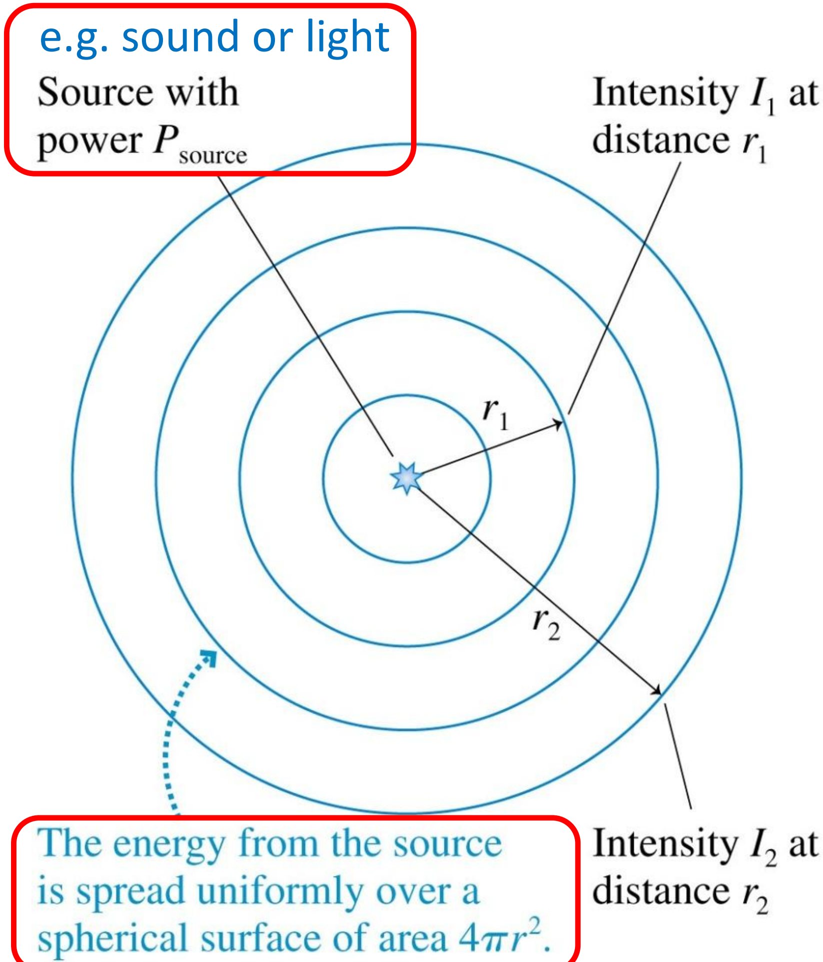


**There are actually two models of light that are used in physics. The Wave Model was discovered by Maxwell in the 1860's (we'll look at this in PHY182); however, in the early 20th century, it was realized that Maxwell's wave theory didn't work in the realm of atoms. As we'll see in a few weeks, this requires that light be viewed as a particle with wave properties or a wave with particle properties – a photon.*

Power and Intensity

Power, P = rate at which the wave transfers energy [units = Watts]

Intensity, I = Power/Area



For a point source (e.g. sound or light) emitting uniformly in Three Dimensions:

The **Intensity at a distance** r from the source is:

$$I = \frac{P_{\text{source}}}{4\pi r^2}$$

[Units = Watts/m^2]

For sound, we define a new unit of sound intensity: **the Decibel (dB):**

$$\text{Sound intensity, } \beta = (10\ dB)\log_{10}\left(\frac{I}{I_0}\right)$$

$$\text{where: } I_0 = 1.0 \times 10^{-12}\ \frac{W}{m^2}$$

There is a similar intensity unit used in astronomy for light intensity, the astronomical magnitude scale – we won't cover it here.

Whiteboard Problem: 16-5

The **Solar Luminosity** is 4.0×10^{26} W. This is the **power** of the electromagnetic waves emitted by the Sun.

a) **What is the intensity of this solar radiation just outside the atmosphere of the Earth? (LC)**

b) ***How much solar energy is available to us on Earth?***
Calculate the solar energy intercepted by the Earth in Joules in one day.
Now, the fun part: how much is one day's amount of available solar energy incident on the Earth worth in dollars. **(LC)**

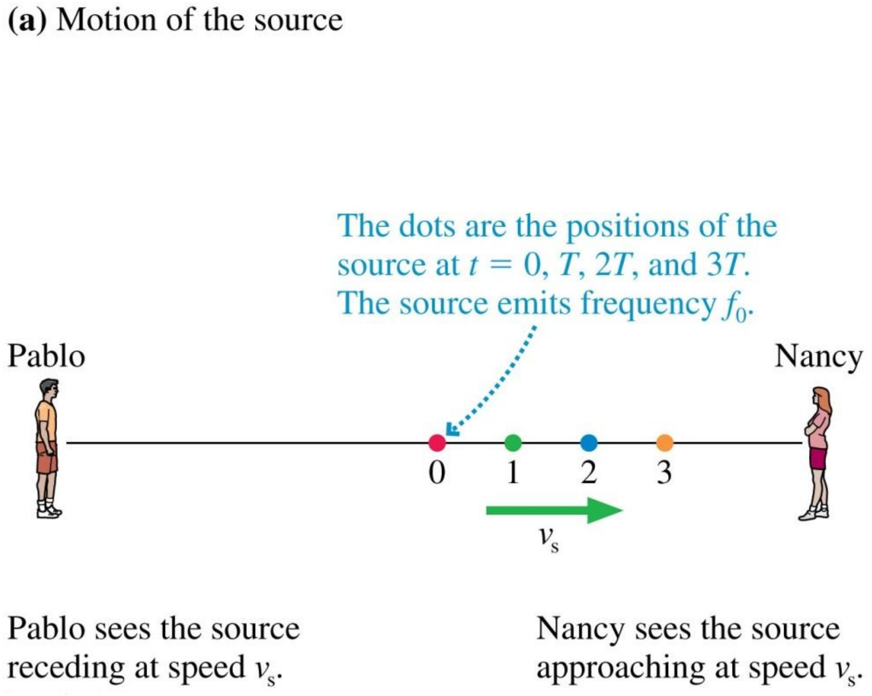
Energy is sold in units of kilowatt-hours (kWh) where 1 kWh = 3.6×10^6 J.
The typical cost of energy is a about 1 dime/kWh, i.e. $0.10/kWh.

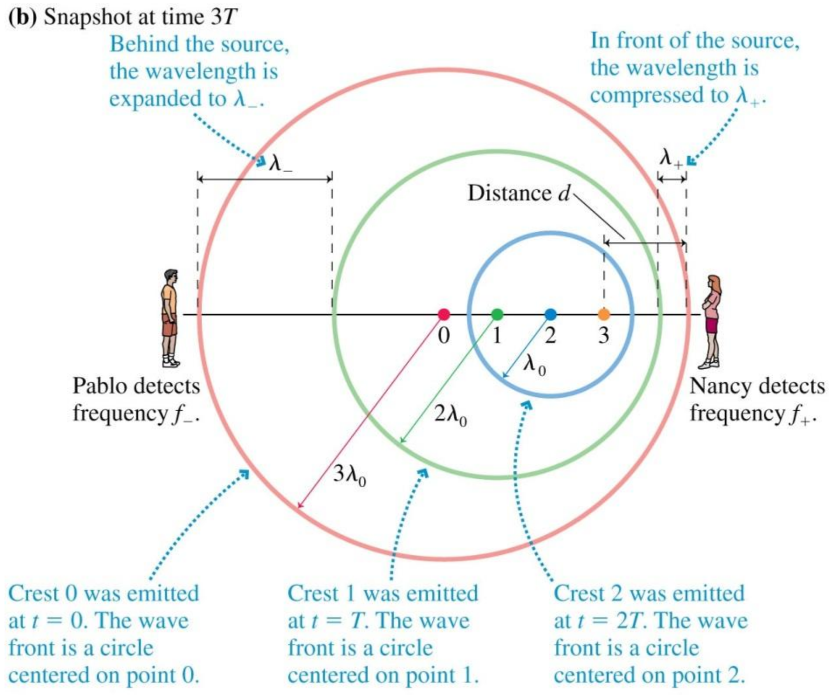
Radius of Earth's orbit, $r_E = 1.5 \times 10^{11}$ m
Radius of Earth, $R_E = 6.37 \times 10^6$ m

The Doppler Effect

The Doppler Effect refers to a change in the perceived wavelength (or frequency) of a wave due to the motion of the source or the observer. It was discovered in the 1840's when it was first noticed for trains.

What causes the Doppler Effect?





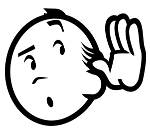
Here's a fun computer demo of the Doppler Effect for Sound

Doppler Effect for Sound

One confusing aspect of the Doppler Effect is that it has a different set of equations for sound than for light. The reason for this is that sound waves pass through a medium such as air; whereas light requires no medium.

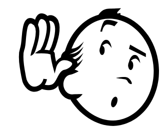
Stationary Observers and a Moving Source:

Hears f_-



$v_{\text{source}} = \text{speed of source relative to medium}$

Hears f_+



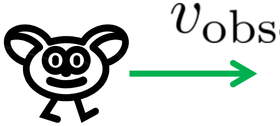
Source emits frequencey f_0 at wave speed v_{sound}

$$f_\pm = \frac{f_0}{\left(1 \mp \frac{v_{\text{source}}}{v_{\text{sound}}}\right)} = f_0 \left(\frac{v_{\text{sound}}}{v_{\text{sound}} \mp v_{\text{source}}} \right)$$

upper signs for f_+

lower signs for f_-

Moving Observers and a Stationary Source:

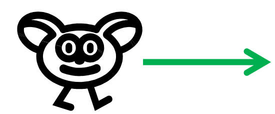


v_{observer}

Hears f_+

Source emits frequencey f_0 at wave speed v_{sound}

v_{observer}



Hears f_-

$$f_\pm = f_0 \left(1 \pm \frac{v_{\text{observer}}}{v_{\text{sound}}} \right) = f_0 \left(\frac{v_{\text{sound}} \pm v_{\text{observer}}}{v_{\text{sound}}} \right)$$

upper signs for f_+

lower signs for f_-

Whiteboard Problem: 16-6

Here's a video of a **demonstration of the Doppler Effect** in sound.

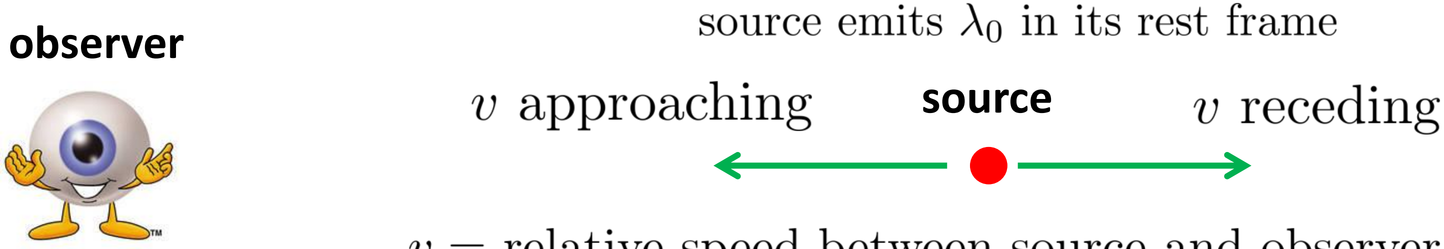
When at rest, the speaker's frequency is 620 Hz. I estimate from the video that the rope is about 1.0 m long and is going around a horizontal circle at about 100 rpm.

What are the highest (LC) and lowest frequencies heard by the students in the classroom?

Doppler Effect for Light

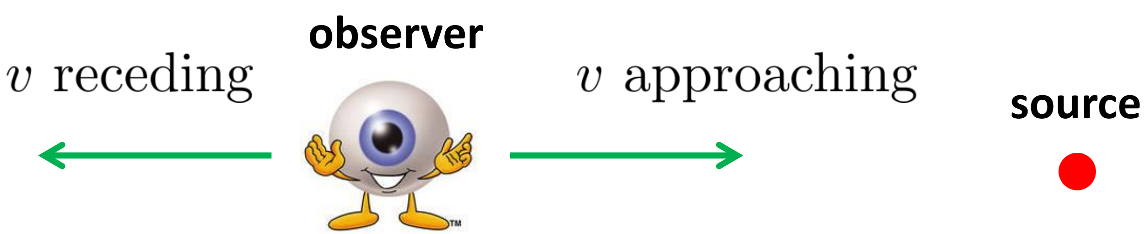
Light waves do not require a medium, so the only speed that is important for the Doppler Effect for light is the **relative speed, v**.

source emits λ_0 in its rest frame



v = relative speed between source and observer

OR, since only relative velocity matters:



For either case, the observed wavelength is:

$$\lambda_{\mp} = \lambda_0 \sqrt{\frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}}}$$

Upper signs for receding

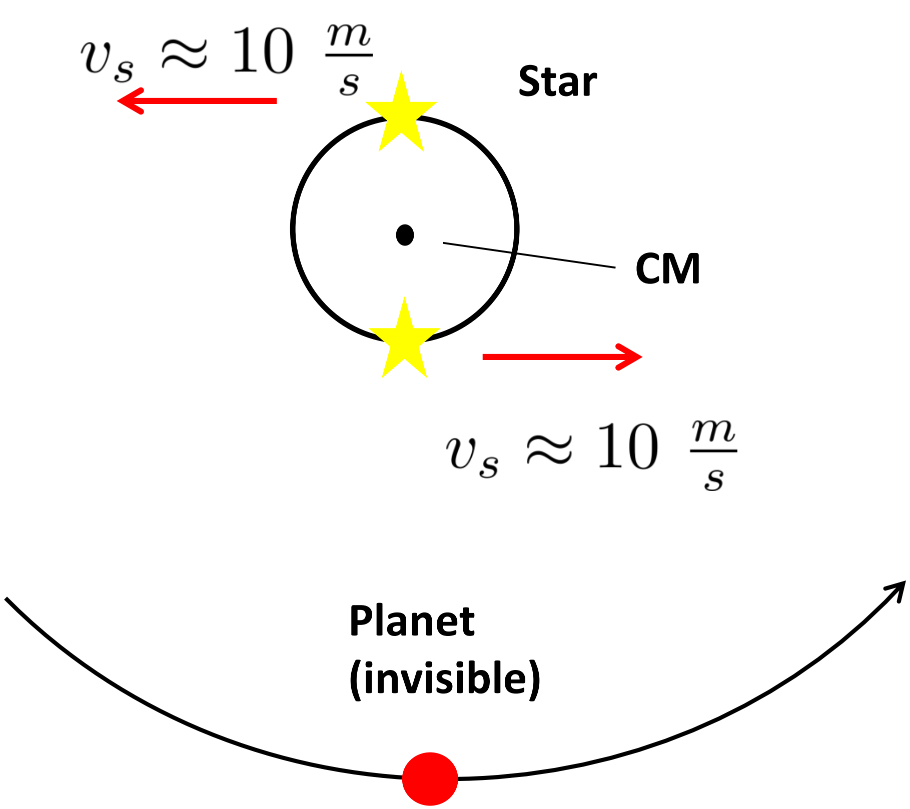
Lower signs for approaching

Whiteboard Problem 16-7: Extrasolar Planets

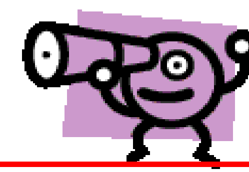
Even with the biggest telescopes, it is impossible to see a planet orbiting another star. Then, how do we detect planets orbiting other stars?

One way to do it is to measure the Doppler shift of the light emitted by the star as the star and planet orbit their common center of mass. A periodic Doppler shift gives the size of the orbit and mass of the planet.

Typical situation:



Us on the Earth



If the star emits wavelength λ_0 when it's at rest, what wavelength do we observe when the star is approaching us and when it is receding?

LC: enter λ/λ_0 for star receding

It's amazing, but this can actually be observed!