

12-3: Rotational Motion – Part 3

A Comparison of Translation and Rotation Equations

Translation

Rotation (about a fixed axis)

Kinematics (1D)

$$v = \frac{dx}{dt} \quad \& \quad a = \frac{dv}{dt}$$

$$\omega = \frac{d\theta}{dt} \quad \& \quad \alpha = \frac{d\omega}{dt}$$

Integrated for Constant Acceleration

$$x_f = x_i + v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

$$v_f = v_i + a \Delta t$$

$$\omega_f = \omega_i + \alpha \Delta t$$

$$v_f^2 = v_i^2 + 2a(\Delta x)^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)^2$$

Dynamics: Newton's 2nd Law

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{\tau}_{net} = I\vec{\alpha}$$

Kinetic Energy

$$K = \frac{1}{2} m v^2$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

Momentum

$$\vec{p} = m\vec{v}$$

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$$\vec{p}_{final} = \vec{p}_{initial}$$

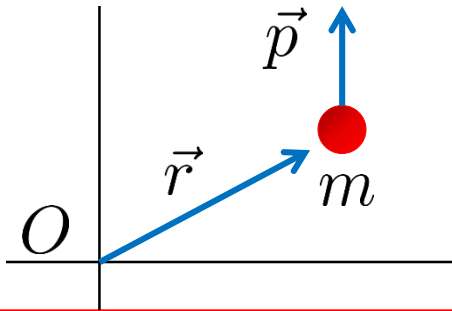
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From the Last Class: Today, we'll finish the table and use this stuff in some tough, but fun, problems.

Angular Momentum

When you have rotational motion, we've seen that you have rotational kinetic energy. We also have the rotational analog of momentum, angular momentum.

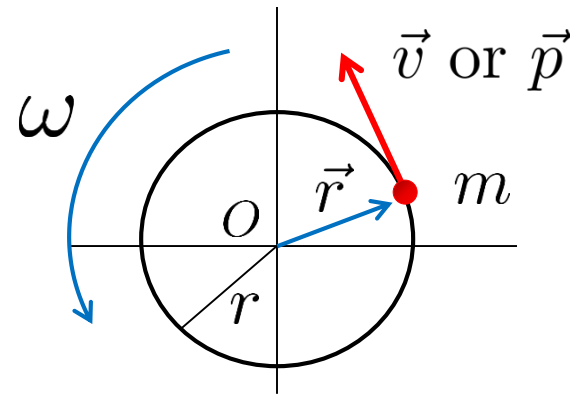
For a particle:



Angular Momentum about Point O :

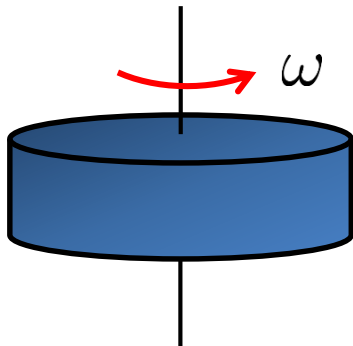
$$\vec{L} = \vec{r} \times \vec{p}$$

Uniform Circular Motion:



$$|\vec{L}| = |\vec{r} \times \vec{p}| = rp = mrv = mr^2\omega$$

Rigid Body rotating about a fixed symmetry axis:



$$\vec{L} = I\vec{\omega} \quad (I = \text{inertia about rotation axis})$$

For here, we'll use the vector sign just to indicate positive for CCW and negative for CW.

Angular Momentum Conservation

Starting with: $\vec{L} = \vec{r} \times \vec{p}$:

$$\begin{aligned}\frac{d\vec{L}}{dt} &= \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} \\ &= \vec{v} \times m\vec{v} + \vec{r} \times \underline{\vec{F}_{net}} \\ &= \vec{\tau}_{net} \text{ (the net torque)}\end{aligned}$$

So, we have another form of Newton's 2nd Law for Rotation:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

Now, if there are no external torques acting on a system, then:

$$\vec{\tau}_{net} = 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L} = \text{Constant}$$

In all of the problems that we'll do, we'll apply conservation of angular momentum like this:

$$\vec{L}_{\text{final}} = \vec{L}_{\text{initial}}$$

And remember, the vector sign just indicates positive for CCW and negative for CW.

We can now finish the table that we started at the beginning



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Integrated for Constant Acceleration

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$$v_f = v_i + a \Delta t$$

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Rotation (about a fixed axis)

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$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

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$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)^2$$

Dynamics: Newton's 2nd Law

$$\vec{F}_{net} = m\vec{a}$$

$$\vec{\tau}_{net} = I\vec{\alpha}$$

Kinetic Energy

$$K = \frac{1}{2} m v^2$$

$$K_{rot} = \frac{1}{2} I \omega^2$$

Momentum

$$\vec{p} = m\vec{v}$$

$$\vec{L} = \vec{r} \times \vec{p} \quad (\text{particle})$$

$$\vec{L} = I\vec{\omega} \quad (\text{rigid body})$$

$$\vec{p}_{final} = \vec{p}_{initial}$$

$$\vec{L}_{final} = \vec{L}_{initial}$$

Whiteboard Problem 12-12

A 2.0 kg, 20 cm diameter turntable rotates at 100 rpm on frictionless bearings. Two 500 g blocks fall from above, hit the turntable simultaneously at the opposite end of a diameter, and stick.

What is the turntable's angular velocity, in rpm, after this event? (LC)

Video WB 12-13: Skaters and Spinning

I'm sure that you all have seen spinning figure skaters. Here's 1976 Olympic gold medalist [Dorothy Hamill](#) sometime in the 1980's or 1990's – I think.

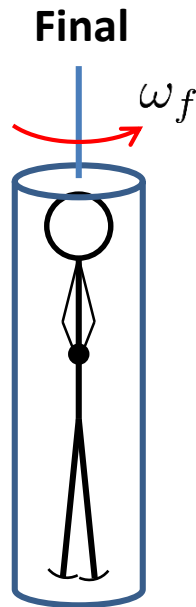
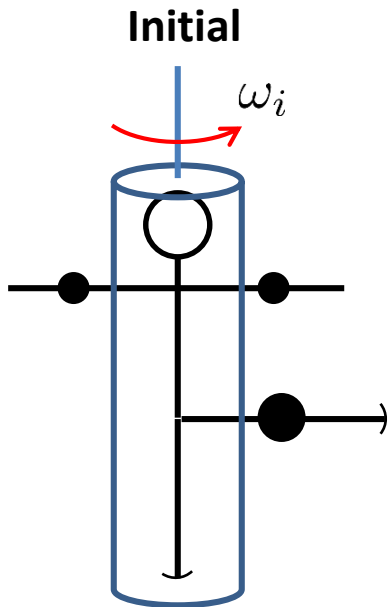
How does Dorothy spin so fast?

As she pulls in her arms and one leg, she is decreasing her moment of inertia, and conservation of angular momentum then says that her spin rate must go up.

Try it on one of our spinning stools.

[Here's Natalia Kanounnikov attempting a world record spin in 2007.](#) (loop, may need VLC)

Suggested Model



$$L_f = I_{\text{cyl}}\omega_f$$

This video is real time. **Try to measure her initial spin rate in rpm (LC).**

Now, make estimates about her size and mass.

I suggest that you treat her body as a cylinder of mass M and radius R and her arms and one leg as point masses located at the centers of mass of the arms and leg.

(a human arm is about 5% of the total body mass and a leg is about 15%)

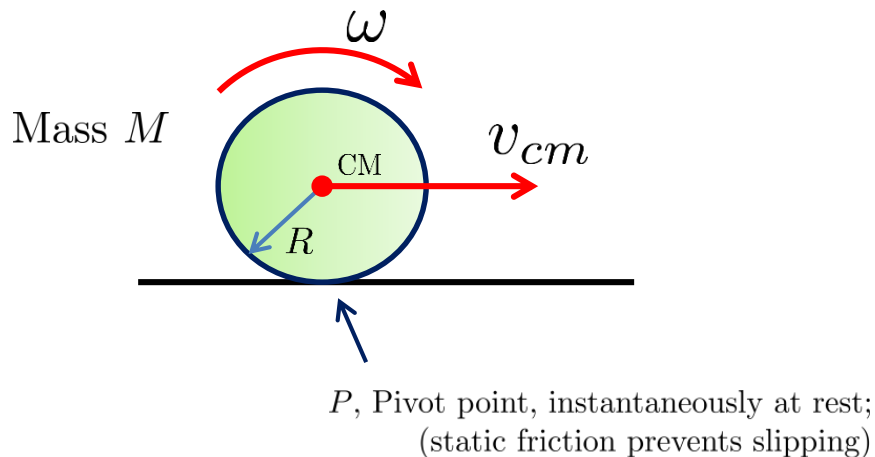
Using your estimates, determine Natalia's final spin rate in rpm. (LC, 2 pt) - also, try out a stool.

[Here's her measured speed.](#)

Rolling Motion

Rolling motion without slipping is a rather complicated motion, but the results of an analysis give a rather simple description that can be applied to many problems.

Your text shows that rolling motion can be described as:



“Translation of the center of mass at v_{cm} and rotation about the center of mass at ω ”

Where v_{cm} and ω are related by:

$$v_{cm} = R\omega$$

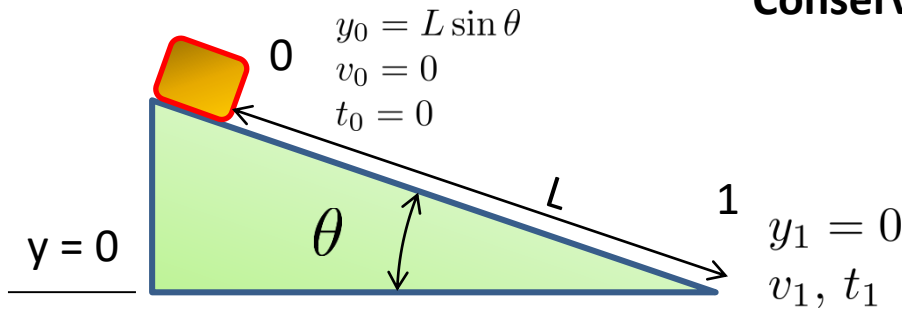
“Rolling Constraint”

We can solve problems using dynamics and/or energy conservation if we now **include the rotational kinetic energy with the translational:**

$$K = K_{\text{trans}} + K_{\text{rot}} = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$$

Whiteboard Problem 12-14: Analysis of Demo Rolling Race

There are several ways to determine the time to get to the bottom **for a body sliding without friction**; here's one:



Conserve energy 0 to 1: $\Delta E_{\text{mech}} = \Delta K + \Delta U_g = 0$

$$\frac{1}{2}m(v_1^2 - v_0^2) + mg(y_1 - y_0) = 0$$

$$\frac{1}{2}v_1^2 - gL \sin \theta = 0 \Rightarrow v_1 = \sqrt{2gL \sin \theta}$$

From 1D Kinematics for x-axis parallel to incline:

$$v_1^2 = v_0^2 + 2a\Delta x \Rightarrow a = \frac{v_1^2}{2L} = \frac{2gL \sin \theta}{2L} = g \sin \theta$$

$$\text{And, } x_1 = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2 \Rightarrow x_1 = L = \frac{1}{2}g \sin \theta t_1^2 \Rightarrow t_1 = \sqrt{\frac{2L}{g \sin \theta}}$$

Your Problem: Find an expression for the time it takes for a rolling object to go down the incline with no rolling friction. **Follow the same steps as above.** Assume that the object has moment of inertia about its center of mass,

$$I_{\text{cm}} = cMR^2 \text{ where } c \text{ is some number}$$

Enter your expression for the time in LC (2 pt, your expression should contain c).

Whiteboard Problem 12-15 (a really fun problem)

Objects of mass m can either slide without friction or roll without slipping down the track that has a loop-the-loop circle of radius R .

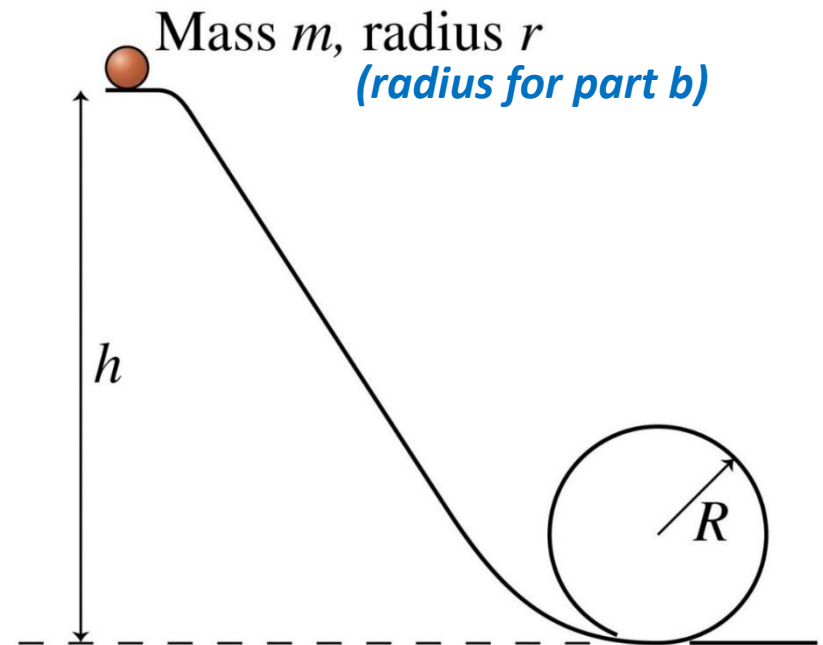
We want to find an expression for the minimum height, h , in terms of the radius R that the track must have for the object to make it around the loop without falling off.

- a) Find the minimum height if the object is a point mass m that slides without friction. **(LC)**

Note: this doesn't involve anything from Chapter 12. Just use what you know from Chapters 9 and 10. Here, your expression should depend just on R .

- b) Now find the minimum height for a marble of mass m and radius r that rolls without slipping. **(LC, 2pt)**

Note: Here, your expression should depend on R and r .

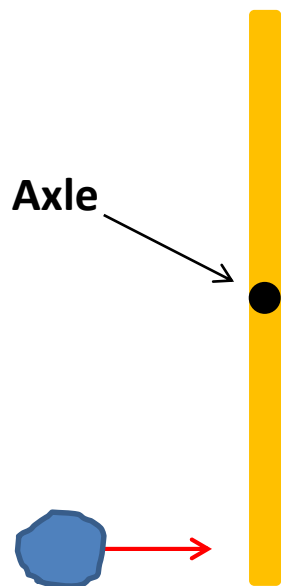


Whiteboard Problem 12-16

This problem is in HW: problem 12.90; use this for hints.

A 30 cm long, 75 g, uniform rod hangs vertically and can rotate on a frictionless axle through its center. A 10 g ball of clay travelling horizontally at 2.5 m/s hits and sticks to the very bottom tip of the rod.

Determine the maximum angle, measured from the vertical, through which the rod (with the attached ball of clay) rotates. (LC, 2 pt)



My sketch:

$L = 30 \text{ cm}$
 $M = 75 \text{ g}$
 $\omega_0 = 0$
 $v_0 = 2.5 \text{ m/s}$
 $m = 10 \text{ g}$
 $r = 15 \text{ cm}$
 ω_1
(both rod & clay)
 $\omega_2 = 0$
 $y_2 = h$
 $y = 0$

Conserve Angular Momentum $0 \rightarrow 1$
Then, conserve energy $1 \rightarrow 2$.