11-2: Quick Review of the Last Class

Definition: **Momentum of the object:** $\vec{p} = m\vec{v}$ Units: $\left[\frac{kg\ m}{s}\right]$

Define: **Impulse** of a Force, $\vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$

Conservation of Momentum:

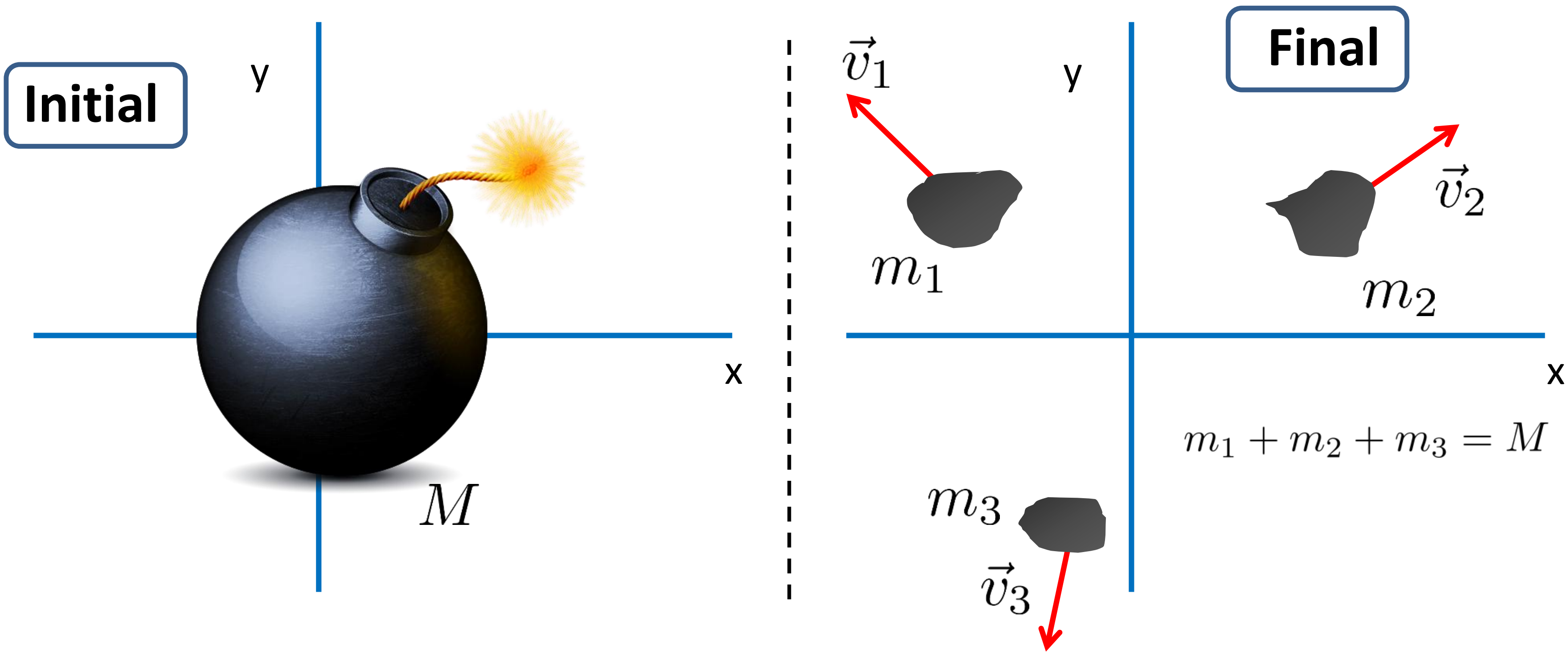
For an Isolated System:

$$\vec{p}_{\text{final}} = \vec{p}_{\text{initial}}$$

A Perfectly Inelastic Collision is a collision where the bodies **stick together and have the same final velocity**. Perfectly Inelastic Collisions conserve momentum.

Explosions

An explosion is just a completely inelastic collision in reverse; like these fireworks. We can use momentum conservation to solve for the velocities of the fragments.



$$\vec{p}_i = 0$$

$$\vec{p}_f = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{p}_i = 0$$

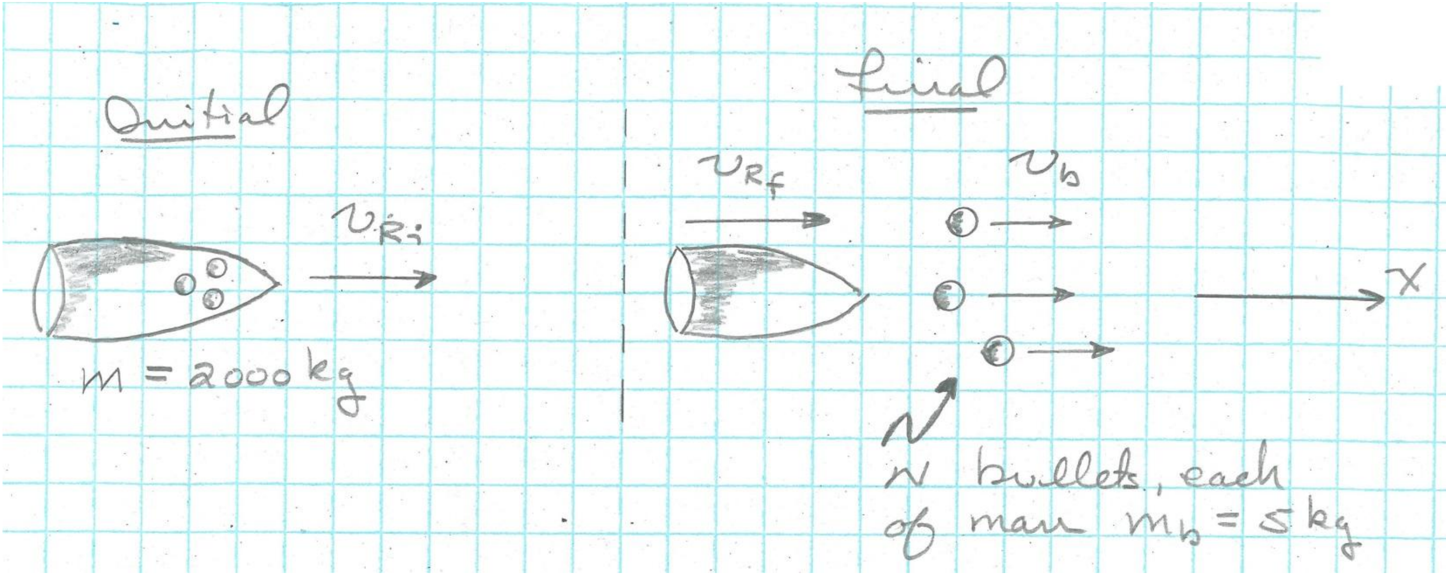
A Real Good Whiteboard Problem 11-6

You are the mission commander of a 2000 kg satellite that is approaching Mars at a speed of 25,000 km/h. It needs to quickly slow to 15,000 km/h to begin its descent to the surface. *If the satellite enters the Martian atmosphere too fast, it will burn up, and if it enters too slowly, it will use up its maneuvering fuel before reaching the surface and will crash.* The satellite has a new braking system: Several 5.0 kg "bullets" on the front that can be fired straight ahead. Each has a high explosive charge that fires it to a speed of 139,000 m/s relative to the satellite. As mission commander, you need to tell it how many bullets to fire – *and oh by the way, this is a $1.5 billion mission, so you want to get it right.*

How many bullets should be fired? (LC)

(Note: you can only fire a whole number of bullets, round your answer up.)

Your sketch should look like this:



PROBLEM-SOLVING STRATEGY 11.1

Conservation of momentum

Remember this:

MODEL Clearly define the *system*.

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or conservation of energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_f = \vec{P}_i$. In component form, this is

$$(p_{fx})_1 + (p_{fx})_2 + (p_{fx})_3 + \cdots = (p_{ix})_1 + (p_{ix})_2 + (p_{ix})_3 + \cdots$$
$$(p_{fy})_1 + (p_{fy})_2 + (p_{fy})_3 + \cdots = (p_{iy})_1 + (p_{iy})_2 + (p_{iy})_3 + \cdots$$

$$p_{x_i} = p_{x_f}$$
$$p_{y_i} = p_{y_f}$$

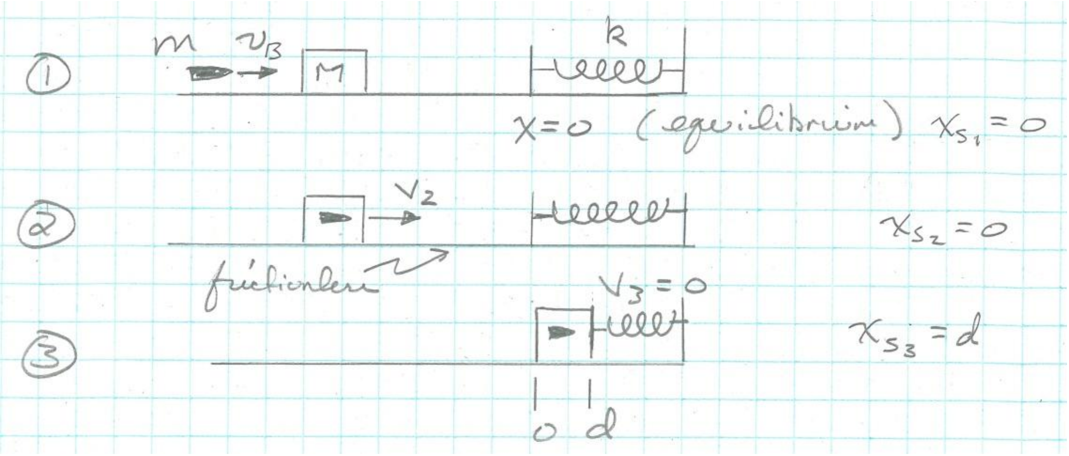
ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

where $\vec{p}$ is the total momentum of the system

Another Really Good Whiteboard Problem 11-7

A *ballistic spring system* is used to measure the speed of bullets. A bullet of mass *m* is fired into a block of wood of mass *M*. The block, with the embedded bullet, then slides across a frictionless table and collides with a horizontal spring of spring constant *k*. The other end of the spring is anchored to the wall. The block compresses the spring a distance *d* before it comes to rest. **Find an expression for the bullet's speed v_b in terms of *m*, *M*, *k*, and *d*.** **(LC)**

Your sketch should look like this:



How do you proceed?

Conserve momentum 1 to 2, and then energy 2 to 3.

Ping Pong Ball Canon Demo and WB Problem 11-8

In this activity, we are going to attempt to determine the speed of a ping pong ball launched from our Ping Pong Ball Canon.

Here's a demo of our canon. And here's a **short video** of how the canon works.

We want to find the speed of the ball as it comes out of the canon.

First, we're going to put some tape on the can so that the collision is completely inelastic and the ball becomes embedded in the can and the combined mess flies off the table and lands on the other side of the room. **What data will we need to calculate the ball's speed before the collision with the can?**

Mass of the ball: m = _____

Height of the table: h = _____

Mass of the can: M = _____

Horizontal distance travelled by can and ball after the collision: d = _____

WB Problem 11-8:

a) **Using the above symbols for the necessary quantities, find an expression for the speed of ball before the collision. (LC, a 2-point shot)**

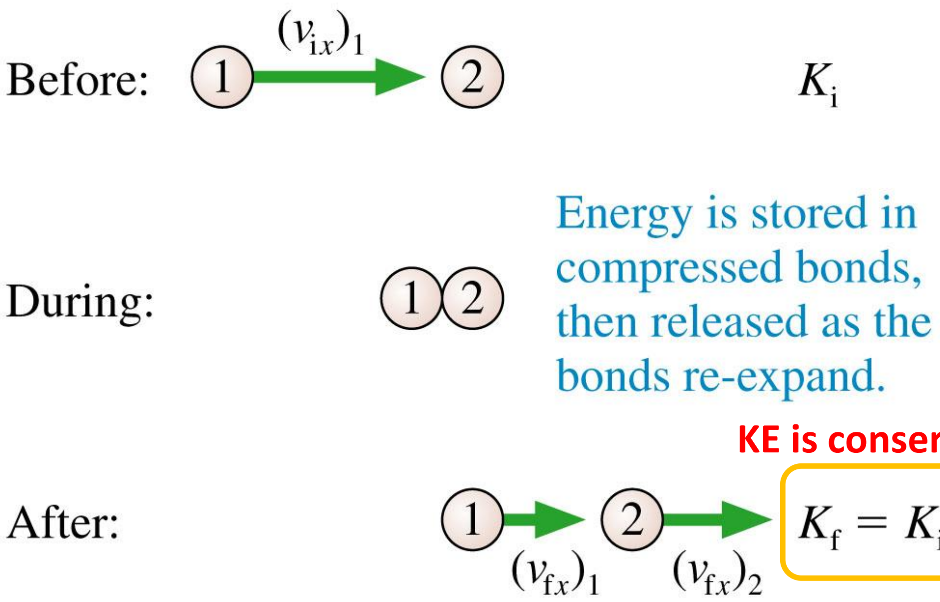
b) **Now, use our data to calculate the speed. (LC)**

c) **Using your speed from part b, how much energy was lost in the collision? (LC) (Where did this energy go?)**

Perfectly Elastic* Collisions

We've been working with **Perfectly Inelastic Collisions** where two objects stick together and have the same final velocity. **These can be solved with momentum conservation alone.**

A **Perfectly Elastic Collision** conserves **both** momentum and mechanical energy.



During the collision, some KE is transferred to PE; then this is transferred back to KE with 100% efficiency.

KE is conserved

$$K_f = K_i$$

Along with momentum:

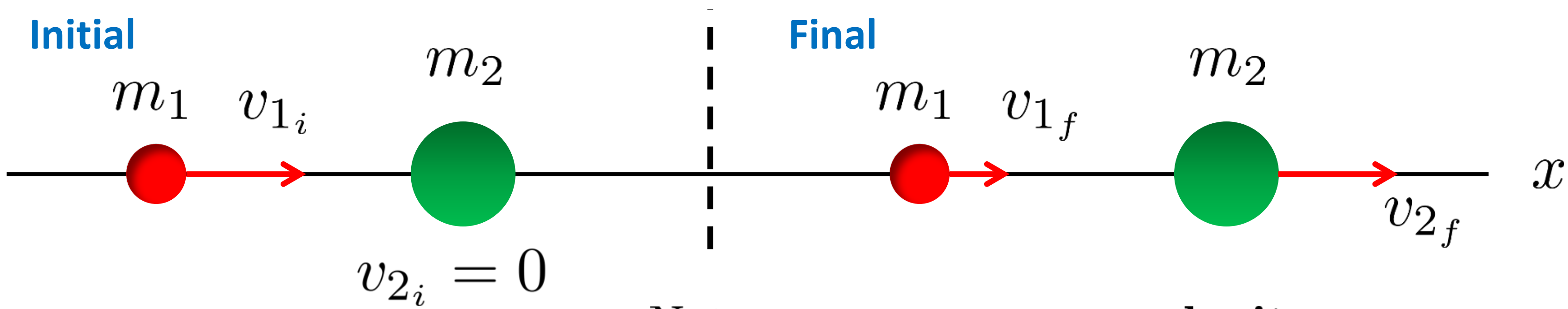
$$\vec{p}_f = \vec{p}_i$$

Note, in real collisions, there is always some energy lost to thermal energy.

*Watch the word **elastic** here. It has nothing to do with the elasticity of a spring. For collisions, it means that the mechanical energy is conserved along with the momentum.

Perfectly Elastic Collisions – A Special Case

Your author considers the Special Case of a **1D perfectly elastic collision between two bodies where one is initially at rest**.



Note: v_{1_i}, v_{1_f}, v_{2_f} are **velocity components**

Using momentum and kinetic energy conservation, the final **velocities** are *(with slight differences in subscripts; these are on the equation sheet)*:

$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1 \qquad (v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1 \qquad \textbf{(11.31)}$$

(perfectly elastic collision with ball 2 initially at rest)

Consider the limiting cases. $m_1 = m_2$; $m_1 >> m_2$; $m_2 >> m_1$

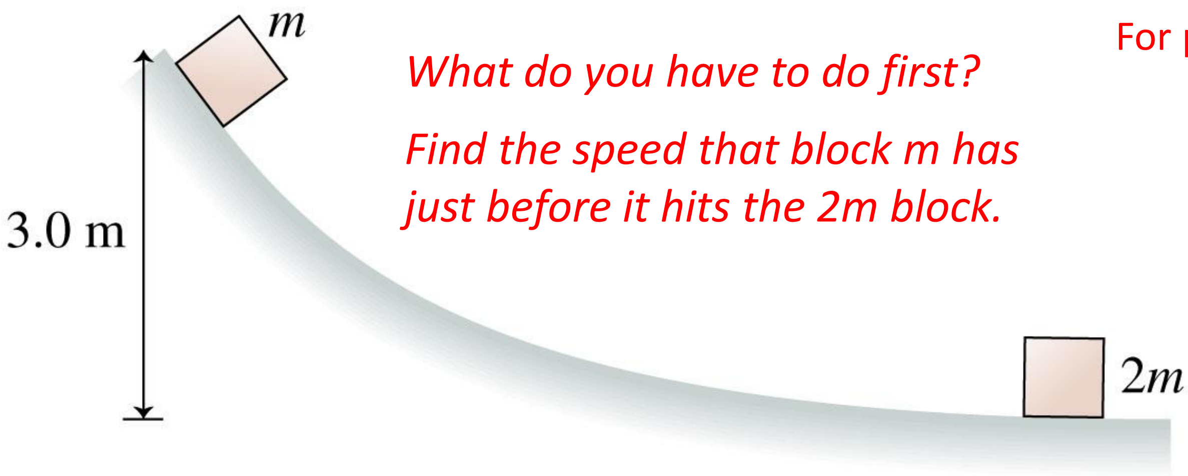
For anyone who is interested, the derivation looks like this . . *it's actually quite fun!*

Whiteboard Problem 11-9

A package of mass *m* is released from rest at a warehouse loading dock and slides down a 3.0 m high frictionless chute to a waiting truck. Unfortunately, the truck driver went on break without having removed the previous package of mass *2m* from the bottom of the chute.

a) **If the packages stick together, what is their speed after the collision? (LC)**

b) **If the collision between the packages is perfectly elastic, to what height does the package of mass *m* rebound? (LC)**



What do you have to do first?

Find the speed that block m has just before it hits the 2m block.

For part b, you might want equ'ns 11-31:

$$v_{1f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1i}$$

$$v_{2f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1i}$$