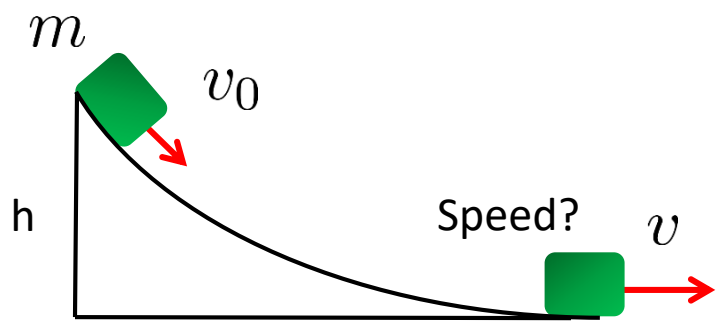
A Quick Recap of Where We Stand

We can now do problems like this:



Energy Conservation:

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{nc}$$

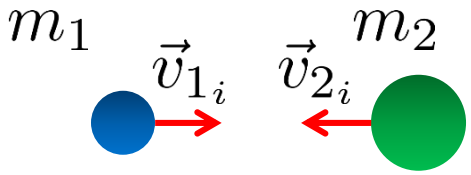
We've looked at two forms of potential energy:

$$\text{Gravity: } U_g = mgy$$

$$\text{Spring: } U_s = \tfrac{1}{2}kx_s^2$$

And we know how to include applied forces and friction.

But we still can't do this, a collision:



Force on m_1 and $m_2 = 0$

$\vec{F}_{2\text{on}1} = -\vec{F}_{1\text{on}2}$

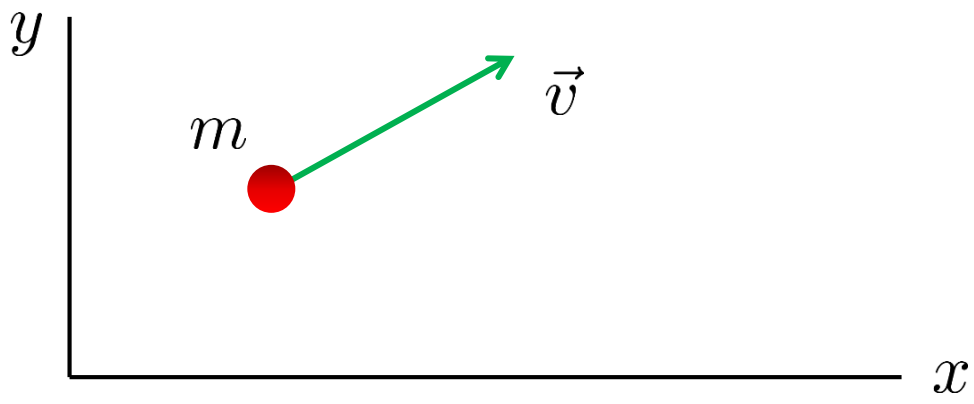
(but, some complicated function of time)

Force on m_1 and $m_2 = 0$

Find: $\vec{v}_{1_f}$ and $\vec{v}_{2_f}$

To solve something like this, we need something new: **Momentum and Impulse**

Momentum and Impulse



Definition: Momentum of the object: $\vec{p} = m\vec{v}$

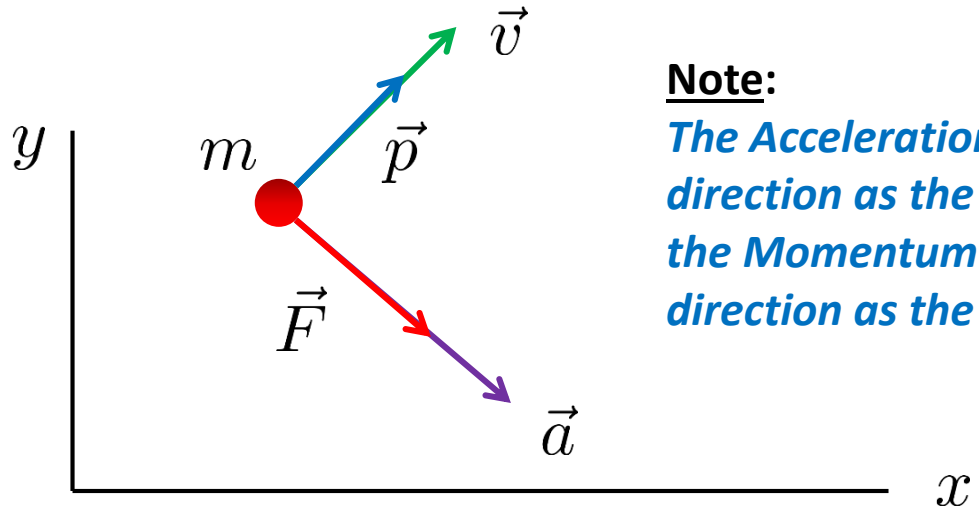
Units: $\left[\frac{kg\ m}{s}\right]$

Note: momentum is a vector quantity, so in most problems, we'll work with it in **component form**:

$$\left.\begin{aligned} p_x &= mv_x \\ p_y &= mv_y \end{aligned}\right\}$$

Can be positive, negative, or zero

Newton's 2nd Law Revisited



Note:
The Acceleration is in the same direction as the Force, and the Momentum is in the same direction as the Velocity.

Newton's 2nd Law:

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \qquad (\text{for } m = \text{constant})$$

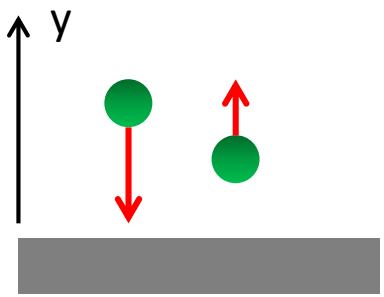
So, another way to write **Newton's 2nd Law** is: $\vec{F} = \dfrac{d\vec{p}}{dt}$

In words: *"Force is the time rate of change of the momentum of the object."*

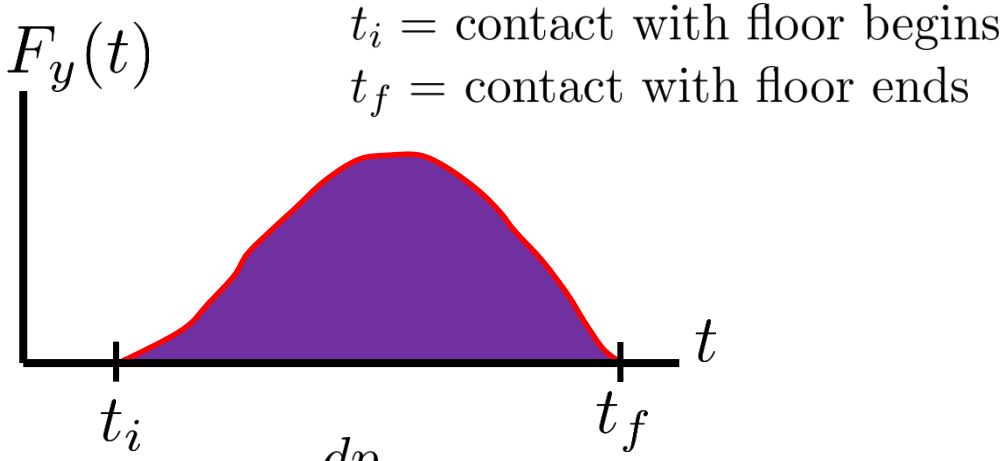
Or even better: ***"A force causes the momentum of the object to change in time."***

Impulse

Suppose an object is subject to a time-varying force: like a ball bouncing from the floor.



The y-component of the contact force on the ball might look something like this:



Apply Newton's 2nd in the y-direction: $F_y(t) = \dfrac{dp_y}{dt}$ or $dp_y = F_y(t)dt$

Integrate both sides: $\displaystyle\int_{p_{y_i}}^{p_{y_f}} dp_y = \int_{t_i}^{t_f} F_y(t)dt$

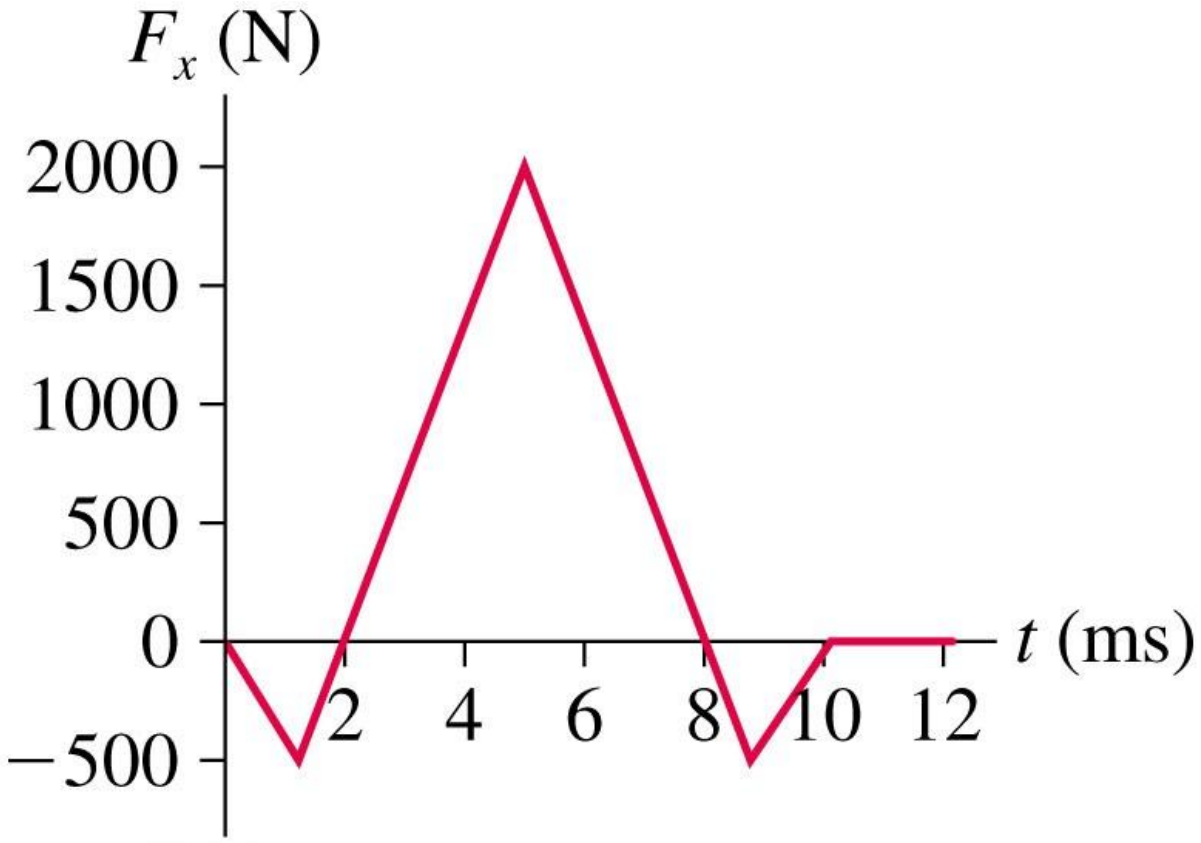
$$p_{y_f} - p_{y_i} = \Delta p_y = \int_{t_i}^{t_f} F_y(t)dt = \text{area under } F_y \text{ vs } t$$

Define: **Impulse** of a Force, $\vec{J} = \Delta\vec{p} = \displaystyle\int_{t_i}^{t_f} \vec{F}(t)dt$

(Describes what a variable force does over time.)

Whiteboard Problem 11-1

What is the impulse on a 3.0 kg particle that experiences the force shown below? (LC)



What do impulsive forces do for us?

We can exert a large force for a brief period of time, like the time that the hammer is colliding with the nail.

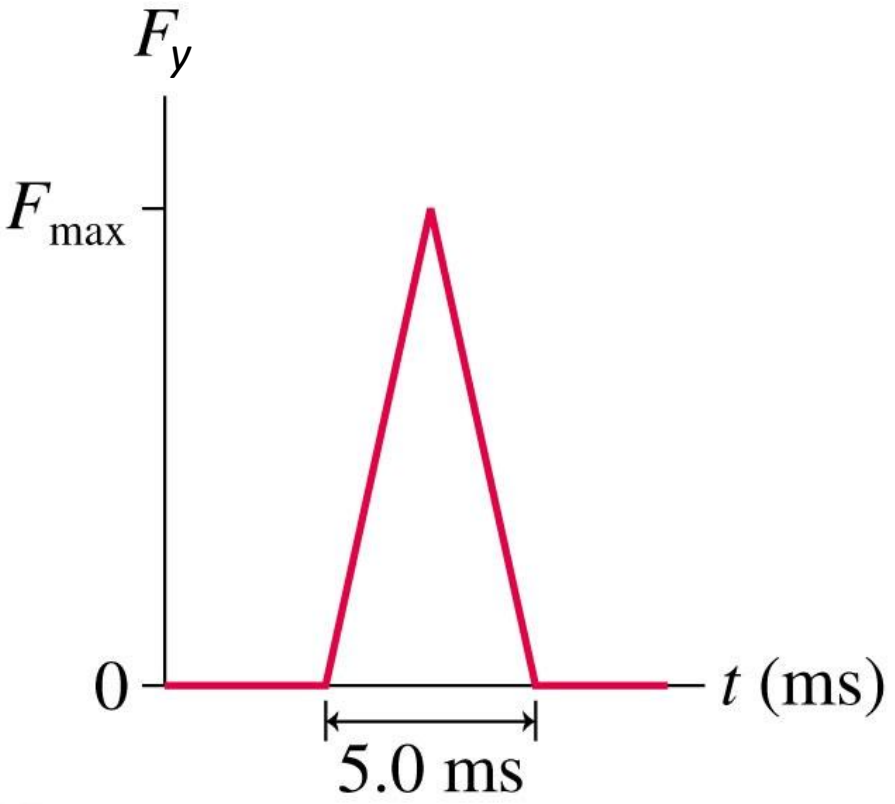
Nail

Whiteboard Problem: 11-2

A 200 g ball is dropped from a height of 2.0 m, bounces on a hard floor, and rebounds to a height of 1.5 m. The figure below shows the impulse received from the floor.

What maximum force, F_{max} , does the floor exert on the ball? (LC)

Hint: *find the velocity of the ball just before it hits the floor and just after it leaves the floor.* ***How?*** *I suggest* ***energy conservation****.*



Whiteboard Problem 11-3: A Diving Board

While relaxing at the Rec Center after studying physics for 10 straight hours, your friend asks you, ***"when someone goes off the diving board, what is the force exerted on the diving board – isn't it just the weight of the diver?"***

You reply, ***"no, the board bends a lot more when he dives than when he just stands on it."***

How can we determine the force the diver exerts on the diving board?

Watch the following film clip of a springboard dive. Assume that the diver has a mass of 60kg. Use the video to estimate the quantities that you need to calculate the maximum force on the diving board – you may assume that the force time profile is triangular like the last whiteboard problem.

a) **What is the maximum force on the diving board? (LC)**

b) **How does this force compare to the weight of the diver?**

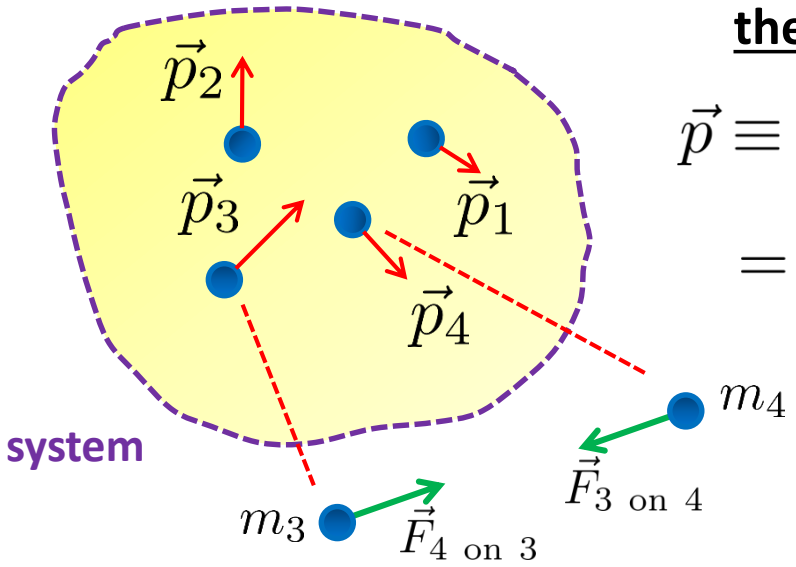
I'll leave the clip on repeat so you can get your measurements – it's in real time.

Conservation of Momentum

Your author shows for a simple two body system and then for a general isolated system of N interacting bodies how momentum of the system is conserved:

For an isolated* system:

***isolated means the net external force on the system is zero**



$$\vec{p} \equiv \text{total momentum of the system}$$

$$= \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots + \vec{p}_N = \sum_{i=1}^{N} \vec{p}_i$$

The forces internal to the system cancel by Newton's 3rd Law.

So for the System:

$$\vec{F}_{\text{net}} = 0 \quad \text{so } \vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} = 0 \quad \text{or } \vec{p} = \text{Constant}$$

Note: we'll almost always apply this to a system before and after an interaction in the form:

For an Isolated System:

$$\vec{p}_{\text{final}} = \vec{p}_{\text{initial}}$$

PROBLEM-SOLVING STRATEGY 11.1

Conservation of momentum

MODEL Clearly define the *system*.

- If possible, choose a system that is isolated ($\vec{F}_{\text{net}} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or conservation of energy.

VISUALIZE Draw a before-and-after pictorial representation. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum: $\vec{P}_{\text{f}} = \vec{P}_{\text{i}}$. In component form, this is

$$(p_{\text{fx}})_1 + (p_{\text{fx}})_2 + (p_{\text{fx}})_3 + \cdots = (p_{\text{ix}})_1 + (p_{\text{ix}})_2 + (p_{\text{ix}})_3 + \cdots$$
$$(p_{\text{fy}})_1 + (p_{\text{fy}})_2 + (p_{\text{fy}})_3 + \cdots = (p_{\text{iy}})_1 + (p_{\text{iy}})_2 + (p_{\text{iy}})_3 + \cdots$$

$$p_{x_i} = p_{x_f}$$
$$p_{y_i} = p_{y_f}$$

ASSESS Check that your result has correct units and significant figures, is reasonable, and answers the question.

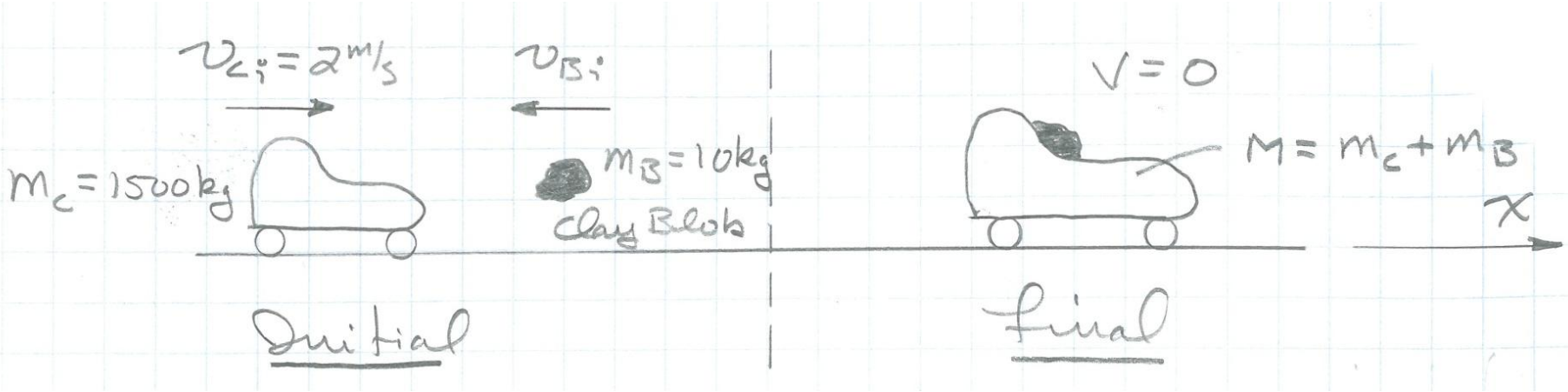
where $\vec{p}$ is the total momentum of the system

Whiteboard Problem 11-4

A 1500 kg car is rolling toward you at 2.0 m/s. You would like to stop the car by firing a 10 kg blob of sticky clay at it.
How fast should you fire the clay; what is its speed? (LC)

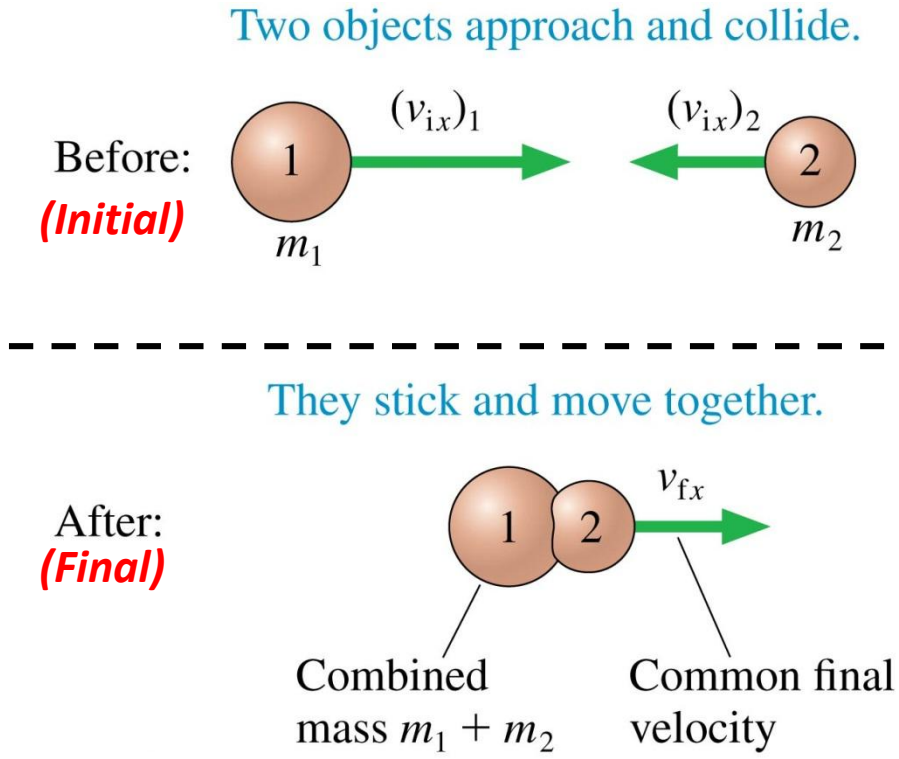
What a ridiculous way to stop a car, but it would work – might break the windshield!

Your sketch should look like this (what is the system?):



Perfectly Inelastic Collisions

The previous problem (WB11-4) is an example of a **Perfectly Inelastic Collision*** which means that the bodies **stick together and have the same final velocity**.



The system is the two balls.

There are no external forces on the system.

Therefore, the momentum of the system is conserved:

$$\vec{p}_{1_i} + \vec{p}_{2_i} = \vec{p}_f$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_f$$

***All collisions conserve momentum**. *For problems where the final velocities are different, we need some more information. We'll see the other extreme, perfectly elastic collisions that also conserve kinetic energy, next class.*

Whiteboard Problem 11-5

(Another perfectly inelastic collision)

A 20 g ball of red clay traveling east at 2.0 m/s collides and sticks to a 30 g ball of blue clay traveling 30° south of west at 1.0 m/s **What are the speed, V, (LC) and direction, ϕ, of the resulting blob of clay?**

Your sketch should look like this: