Physics 181 Equation Sheet for Exams

- **Chapter 1:** Conceptual, no equations

- **Chapter 2:** 1D Motion along the s-axis:
  Average: $v_{s_{\text{avg}}} = \frac{\Delta s}{\Delta t}$ $a_{s_{\text{avg}}} = \frac{\Delta v_s}{\Delta t}$ Instantaneous: $v_s = \frac{ds}{dt}$ $a_s = \frac{dv_s}{dt}$
  Position from velocity: $s_f = s_i + \int_{t_i}^{t_f} v_s dt$ Velocity from acceleration: $v_{s_f} = v_{s_i} + \int_{t_i}^{t_f} a_s dt$
  A Useful derivative and integral: $\frac{d}{dt}(ct^n) = cnt^{(n-1)}$ $\int_{t_i}^{t_f} ct^n dt = c\left[\frac{t^{(n+1)}}{n+1}\right]_{t_i}^{t_f} = \frac{c}{n+1}\left\{t_f^{n+1} - t_i^{n+1}\right\}$
  (c and n are constants)
  Constant Acceleration: $s_f = s_i + v_{s_i}\Delta t + \frac{1}{2}a_s\Delta t^2$ $v_{s_f} = v_{s_i} + a_s\Delta t$ $v_{s_f}^2 = v_{s_i}^2 + 2a_s\Delta s$
  . where: $\Delta t = t_f - t_i$ $\Delta s = s_f - s_i$
  Free Fall: $a_y = -g$ (if $+y$ is up); Inclined Plane: $a_x = g\sin\theta$ (if $+x$ is down incline)

- **Chapter 3:**
  Vector $\vec{A}$ (θ is CCW from +x axis to $\vec{A}$): $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}$ $A_x = A\cos\theta$ $A_y = A\sin\theta$
  . $A = \sqrt{A_x^2 + A_y^2}$ $\theta = tan^{-1}(\frac{A_y}{A_x})$
  Vector Addition: $\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}$

- **Chapter 4:** 2D Kinematics:
  $\vec{v} = \frac{d\vec{r}}{dt}$ $\vec{a} = \frac{d\vec{v}}{dt}$ Components: $v_x = \frac{dx}{dt}$ $v_y = \frac{dy}{dt}$ $a_x = \frac{dv_x}{dt}$ $a_y = \frac{dv_y}{dt}$
  Constant Acceleration: use equations from chap 2 with $s \to x$ and $s \to y$ with a_x and a_y constant
  Projectile Motion: use constant acceleration with $a_x = 0$ and $a_y = -g$ (if up is +y)
  . Range Equation (make sure it applies to your problem) $R = \frac{{v_0}^2}{g}\sin(2\theta)$
  Relative Motion (Galilean Transformations): $\vec{r} = \vec{r'} + \vec{V}t$ $\vec{v} = \vec{v'} + \vec{V}$
  Uniform Circular Motion: $s = r\theta$ $T = \frac{2\pi}{\omega}$ $f = \frac{1}{T}$ $\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i}$ $\omega = \frac{d\theta}{dt}$ $v = r\omega$ $a_r = \frac{v^2}{r}$

- **Chapters 5 and 6:**
  Newton's Second Law: $\vec{F}_{net} = m\vec{a}$ Components: $\sum F_x = ma_x$ $\sum F_y = ma_y$
  . Equilibrium $\Leftrightarrow$ $a_x = 0$ and $a_y = 0$
  Friction: Kinetic: $\vec{f_k} = (\mu_k n$, opposite motion) Static: $\vec{f_s} \leq (\mu_s n$, opposite impending motion)
  . Rolling: $\vec{f_r} = (\mu_r n$, opposite motion)
  Aerodynamic Drag: $\vec{D} = (\frac{1}{2}C\rho Av^2$, opposite $\vec{v}$)

- **Chapter 7:**
  No new equations – know Newton's third law and action/reaction pairs.

- **Chapter 8:**
  2D Dynamics: 2D kinematics and $\vec{F} = m\vec{a}$ in component form
  Uniform Circular Motion: $\vec{F}_{\text{centripetal}} = \left(\frac{mv^2}{r},\ \text{toward center}\right)$

- **Chapters 9 and 10:**
  Kinetic Energy: $K = \frac{1}{2}mv^2$ Gravitational Potential Energy: $U_g = mgy$
  Spring Potential Energy: $U_s = \frac{1}{2}kx_s^2$ ($x_s = 0$ is the equilibrium position of the spring)
  Vector Dot Product: $\vec{A} \cdot \vec{B} = AB\cos\theta = A_xB_x + A_yB_y$
  Work: General: $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{s}$
  . Variable force, linear displacement: $W = \int_{s_i}^{s_f} F_s ds$
  . Constant force, linear displacement: $W = \vec{F} \cdot \Delta\vec{r}$
  Work Energy Theorem: $\Delta K = W_{\text{net}}$
  Potential Energy of a Conservative Force, $\vec{F_c}$: $\Delta U = U_{\text{final}} - U_{\text{initial}} = -W_{i \to f}(\text{by } \vec{F_c})$
  Conservation of Mechanical Energy: $\Delta E_{\text{mech}} = \Delta K + \Delta U = W_{\text{nc}}$
  Force from PE: $F_x = -\frac{dU}{dx}$, $F_y = -\frac{dU}{dy}$, *etc.*
  Power: $P = \frac{dW}{dt} = \vec{F} \cdot \vec{v}$

- **Chapter 11:**
  Momentum: $\vec{p} = m\vec{v}$ Newton's Second Law: $\vec{F} = \frac{d\vec{p}}{dt}$
  Impulse: $\vec{J} = \Delta\vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}} = \int_{t_i}^{t_f} \vec{F}(t)dt$
  Conservation of Momentum: For $\vec{F}_{\text{external}} = 0$, $\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$
  1D perfectly elastic collision (m_1 moving, m_2 motionless): $v_{1_f} = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] v_{1_i}$ $v_{2_f} = \left[\frac{2m_1}{m_1 + m_2}\right] v_{1_i}$

- **Chapter 12:**
  Kinematics: $\omega = \frac{d\theta}{dt}$ $\alpha = \frac{d\omega}{dt}$ $v_t = r\omega$ $a_t = r\alpha$
  Constant α: $\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$ $\omega_f = \omega_i + \alpha \Delta t$ $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
  . where: $\Delta t = t_f - t_i$ $\Delta\theta = \theta_f - \theta_i$
  Center of Mass: $x_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$ $y_{cm} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i$
  Vector Cross Product: $\vec{C} = \vec{A} \times \vec{B} = (AB\sin\theta, \text{direction by RHR})$ (determinant - see below)
  Torque: $\vec{\tau} = \vec{r} \times \vec{F} = (rF\sin\theta, \text{direction by RHR}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$
  $= \hat{\imath}(yF_z - zF_y) - \hat{\jmath}(xF_z - zF_x) + \hat{k}(xF_y - yF_x)$
  Dynamics: $\tau_{\text{net}} = I\alpha$ $I(\text{discrete}) = \sum_{i=1}^{N} m_i r_i^2$ $I(\text{continuous}) = \int r^2 dm$
  Parallel Axis Theorem: $I = I_{cm} + Md^2$
  Rigid Body Equilibrium: $\sum F_x = \sum F_y = \sum F_z = 0$ and $\sum_{\text{any pt.}} \tau = 0$
  Angular Momentum: General: $\vec{L} = \vec{r} \times \vec{p}$ Rigid Body: $\vec{L} = I\vec{\omega}$ Conservation: $\vec{L}_f = \vec{L}_i$
  Rolling Motion: Constraint: $v_{cm} = R\omega$ Total Kinetic Energy, $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

- **Chapter 13:**
  Gravitational Force: $\vec{F}_{1 \text{ on } 2} = -\vec{F}_{2 \text{ on } 1} = \left(\frac{Gm_1m_2}{r^2}, \text{ attractive}\right)$ Acceleration: $g(r) = \frac{GM}{r^2}$
  Potential Energy: $U = -\frac{Gm_1m_2}{r}$ Circular Orbit: $v_{\text{circ}} = \sqrt{\frac{GM}{r}}$ Kepler's 3rd: $T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$

- **Chapter 15:**
  Restoring Force: $F_x = -kx$ Differential equation: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$
  Solution: $x(t) = A\cos(\omega t + \phi_0)$ $\omega = \sqrt{\frac{k}{m}}$ $T = \frac{2\pi}{\omega}$ $f = \frac{\omega}{2\pi} = \frac{1}{T}$
  Energy: $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}mA^2\omega^2$
  UCM Tool: "*ϕ_0 is the angle where the equivalent UCM object is at $t = 0$*".
  Simple Pendulum: $\omega = \sqrt{\frac{g}{L}}$ $T = 2\pi\sqrt{\frac{L}{g}}$ Physical Pendulum: $\omega = \sqrt{\frac{Mg\ell}{I_p}}$ $T = 2\pi\sqrt{\frac{I_p}{Mg\ell}}$
  Damped Oscillations: $x(t) = A_0 e^{-t/2\tau}\cos(\omega t + \phi_0) = A(t)\cos(\omega t + \phi_0)$
  . $\omega = \sqrt{\omega_0^2 - \frac{b^2}{4m^2}}$ $\omega_0 = \text{undamped frequency} = \sqrt{\frac{k}{m}}$ $\tau = \frac{m}{b}$

- **Chapter 16:**

  Wave on a string: $v = \sqrt{\frac{T_s}{\mu}}$

  Sinusoidal Traveling Waves: $D(x,t) = A\sin(kx \mp \omega t + \phi_0) \begin{pmatrix} +x \text{ direction} \\ -x \text{ direction} \end{pmatrix}$

  . where: $f = \frac{\omega}{2\pi} = \frac{1}{T}$ $k = \frac{2\pi}{\lambda}$ $v = \frac{\lambda}{T} = \lambda f = \frac{\omega}{k}$

  Intensity of point source: $I = \frac{P}{4\pi r^2}$ Decibel scale: $\beta = (10\ dB)\log_{10}(\frac{I}{I_0})$ $I_0 = 1 \times 10^{-12}\ \frac{W}{m^2}$

  Index of Refraction: $n = \frac{\text{speed in vacuum}}{\text{speed in medium}} = \frac{c}{v}$

  Doppler Effect for Sound: $\begin{pmatrix} \text{upper signs} \Rightarrow \text{approaching} \\ \text{lower signs} \Rightarrow \text{receding} \end{pmatrix}$

  . Source Moving: $f_\pm = \frac{f_0}{(1 \mp \frac{v_{\text{source}}}{v_{\text{sound}}})} = f_0 \left(\frac{v_{\text{sound}}}{v_{\text{sound}} \mp v_{\text{source}}} \right)$

  . Observer Moving: $f_\pm = f_0 \left(1 \pm \frac{v_{\text{observer}}}{v_{\text{sound}}} \right) = f_0 \left(\frac{v_{\text{sound}} \pm v_{\text{observer}}}{v_{\text{sound}}} \right)$

  Doppler Effect for Light: $\begin{pmatrix} v = \text{relative speed} \\ \text{upper signs} \Rightarrow \text{receding} \\ \text{lower signs} \Rightarrow \text{approaching} \end{pmatrix}$ $\lambda_\mp = \lambda_0 \sqrt{\frac{1 \pm \frac{v}{c}}{1 \mp \frac{v}{c}}}$

- **Chapter 17:**

  Standing Waves: General: $D(x,t) = 2a \sin kx \cos \omega t$

  . On a string: $\lambda_m = \frac{2L}{m}$ $f_m = m\frac{v}{2L}$ $m = 1, 2, 3, \cdots$

  . Sound tubes: closed-closed $\lambda_m = \frac{2L}{m}$ $f_m = m\frac{v}{2L}$ $m = 1, 2, 3, \cdots$

  . Sound tubes: Open-open $\lambda_m = \frac{2L}{m}$ $f_m = m\frac{v}{2L}$ $m = 1, 2, 3, \cdots$

  . Sound tubes: Open-closed $\lambda_m = \frac{4L}{m}$ $f_m = m\frac{v}{4L}$ $m = 1, 3, 5, \cdots$

  Interference: $\Delta\phi = 2\pi\frac{\Delta r}{\lambda} + \Delta\phi_0 = \begin{Bmatrix} m2\pi \text{ (constructive)} \\ (m + 1/2)2\pi \text{ (destructive)} \end{Bmatrix}$ $m = 0, 1, 2, 3, \cdots$

  Beats: $D = 2a\cos(\omega_{\text{mod}}t)\sin(\omega_{\text{avg}}t)$ $\omega_{\text{mod}} = \frac{1}{2}(\omega_1 - \omega_2)$ $\omega_{\text{avg}} = \frac{1}{2}(\omega_1 + \omega_2)$ $f_{\text{beat}} = f_1 - f_2$

- **Quantum Mechanics:**

  Blackbody Radiation: Stefan–Boltzmann Law, Intensity, I (W/m^2) $= \sigma T^4$

  . Wien Displacement Law: $\lambda_{\text{peak}}T = \text{constant} = 2.9 \times 10^6\ nm\ K$

  Photons: $E = hf = \frac{hc}{\lambda}$ $p = \frac{E}{c}$

  Coulomb Force: $F_E = \frac{Kq_1q_2}{r^2}$ $U_E = \frac{Kq_1q_2}{r}$

  Bohr Atom: $r_n = n^2 a_B$ $a_B = 0.0529\ nm$ $v_n = \frac{\hbar}{nma_B}$ $E_n = \frac{-13.6\ eV}{n^2}$ $n = 1, 2, 3, \cdots$

  deBroglie Waves: $\lambda = \frac{h}{p}$ Heisenberg Uncertainty Principle: $\Delta x \Delta p_x \geq \frac{h}{2}$

  Time Independent Schrödinger Equation: $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$

  1D Probability Density: Probability of finding particle at x in width $dx = |\psi(x)|^2 dx$

  Probability of finding the particle between $x = a$ and $x = b$: $P(a \leq x \leq b) = \int_a^b |\psi(x)|^2 dx$

  Most Probable value of x: $x_{\text{mp}} = x$ for $|\psi(x)|^2$ maximum

  Expectation value of x: $\langle x \rangle = \int_{-\infty}^{+\infty} x|\psi(x)|^2 dx$

  Infinite Square Well: $E_n = \frac{n^2h^2}{8mL^2} = \frac{\hbar^2 n^2 \pi^2}{2mL^2}$ $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$ $n = 1, 2, 3, \cdots$

  Tunneling: $P_{\text{tunnel}} = e^{-2w/\eta}$ $\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$

  Simple Harmonic Oscillator: $E_n = (n - 1/2)\hbar\omega$ $n = 1, 2, 3, \cdots$

  Hydrogen Atom: $E_n = \frac{-13.6\ eV}{n^2}$ $n = 1, 2, 3, \cdots$

  . $|\vec{L}| = \sqrt{\ell(\ell + 1)}\hbar$ $\ell = 0, 1, 2, \cdots, (n - 1)$

  . $L_z = m\hbar$ $m = -\ell \rightarrow +\ell$ in integer steps