# The Mathematical Principles of Natural Philosophy (1846)/BookI-III

This is Newton's proof that elliptical orbits imply an inverse square law of gravity.



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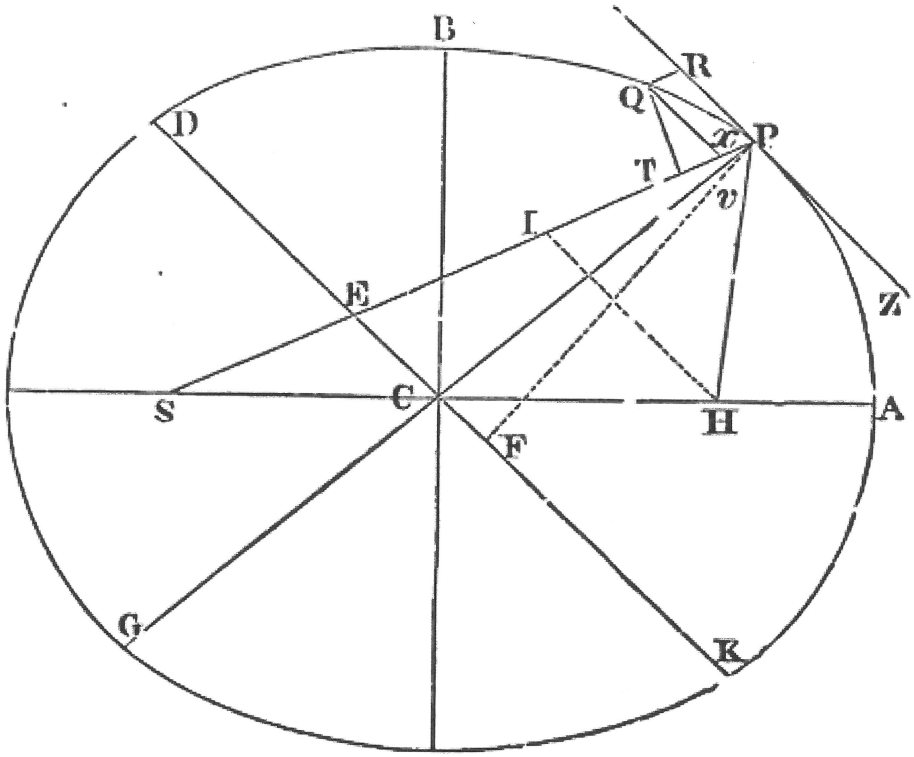
## SECTION III.

*Of the motion of bodies in eccentric conic sections.*

### PROPOSITION XI. PROBLEM VI.

*If a body revolves in an ellipsis; it is required to find the law of the centripetal force tending to the focus of the ellipsis.*




Let S be the focus of the ellipsis. Draw SP cutting the diameter DK of the ellipsis in E, and the ordinate Q*v* in *x*; and complete the parallelogram Q*x*PR. It is evident that EP is equal to the greater semi-axis AC: for drawing HI from the other focus H of the ellipsis parallel to EC, because CS, CH are equal, ES, EI will be also equal; so that EP is the half sum of PS, PI, that is (because of the parallels HI, PR, and the equal angles IPR, HPZ), of PS, PH, which taken together are equal to the whole axis 2AC. Draw QT perpendicular to SP, and putting L for the principal latus rectum of the ellipsis (or for $\frac{2BC^2}{AC}$), we shall have $L \times QR$ to $L \times Pv$ as QR to P*v*, that is, as PE or AC to PC; and $L \times Pv$ to G*v*P as L to G*v*; and G*v*P to $Qv^2$ as $PC^2$ to $CD^2$; and by (Corol. 2, Lem. VII) the points Q and P coinciding, $Qv^2$ is to $Qx^2$ in the ratio of equality; and $Qx^2$ or $Qv^2$ is to $QT^2$ as $EP^2$ to $PF^2$, that is, as $CA^2$ to $PF^2$, or (by Lem. XII) as $CD^2$ to $CB^2$. And compounding all those ratios together, we shall have $L \times QR$ to $QT^2$ as $AC \times L \times PC^2 \times CD^2$, or $2CB^2 \times PC^2 \times CD^2$ to $PC \times Gv \times CD^2 \times CB^2$, or as 2PC to G*v*. But the points Q and P coinciding, 2PC and G*v* are equal. And therefore the quantities $L \times QR$ and $QT^2$, proportional to these, will be also equal. Let those equals be drawn into $\frac{SP^2}{QR}$, and $L \times SP^2$ will become equal to $\frac{SP^2 \times QT^2}{QR}$. And therefore (by Corol. 1 and 5, Prop. VI) the centripetal force is reciprocally as $L \times SP^2$, that is, reciprocally in the duplicate ratio of the distance SP. Q.E.I.